Fixed-Point Aspects of MIMO OFDM Detection on SDR Platforms

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Integrierte Systeme der Signalverarbeitung

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Motivation of Software Defined Radio

- **MIMO OFDM Application**
- **Platform Solutions**
  - Exploiting Data Level Parallelism
  - The P2012 Platform
- **Fixed Point Aspects of MIMO Detection**
  - Problem & Mitigation (QR Decomposition)
  - Algorithmic Performance
  - Execution Time
- **Summary & Outlook**

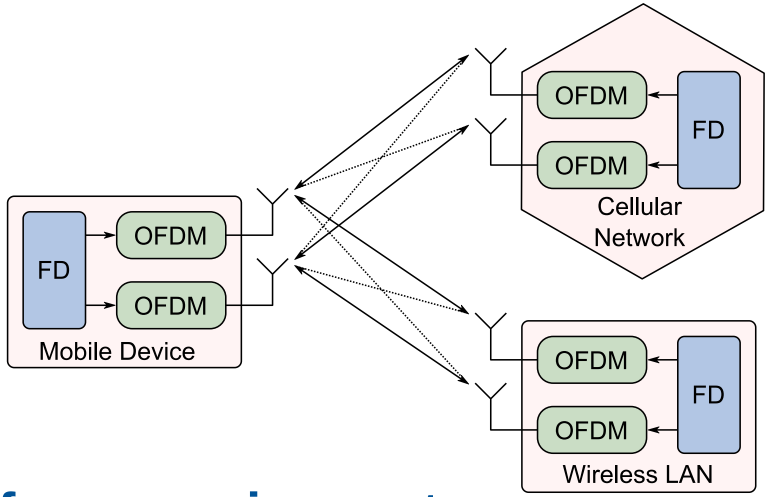
Motivation of Software Defined Radio

- **Modern wireless communication**
  - Wireless LANs (stationary)
    - IEEE 802.11 a/b/g/n
  - Cellular networks (mobile)
    - GSM
    - UMTS
    - LTE
    - Cdma2000
- **Merging of stationary & mobile communication**
  - You expect your …

    … smartphone to also support wireless LAN

    … laptop to also support cellular networks
- **Need for a flexible, programmable platform**
  - *Software Defined Radio*

- **Characteristics of wireless standards (LTE, 802.11n)**
  - High data rates, low latencies
  - *MIMO:* Multiple antenna transmission
  - *OFDM:* Orthogonal frequency-division multiplexing



- **SDR platform requirements**
  - *Multi-core*: Handle high throughput, exploit DLP
    - *Common solutions: SIMD, VLIW*
  - *Fast signaling*: Handle low latency

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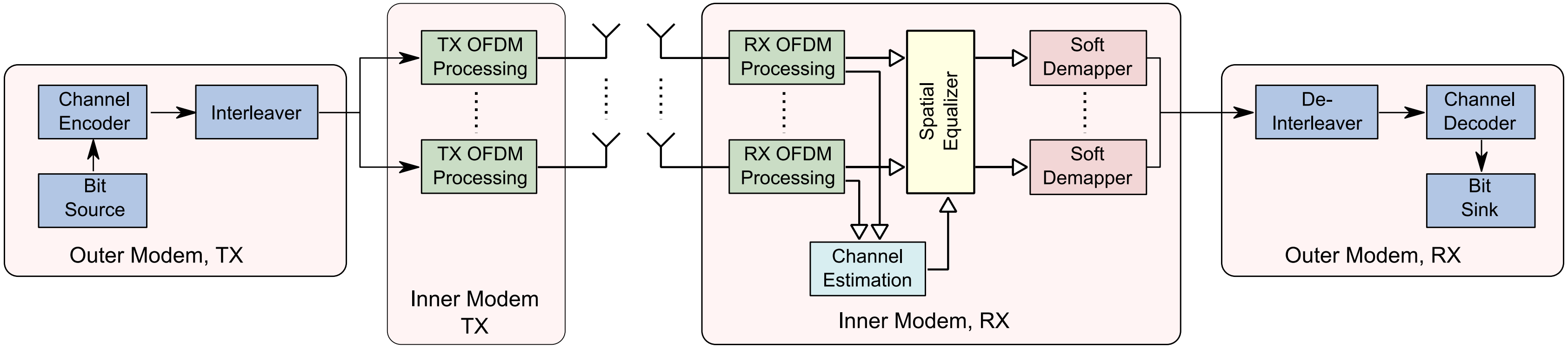
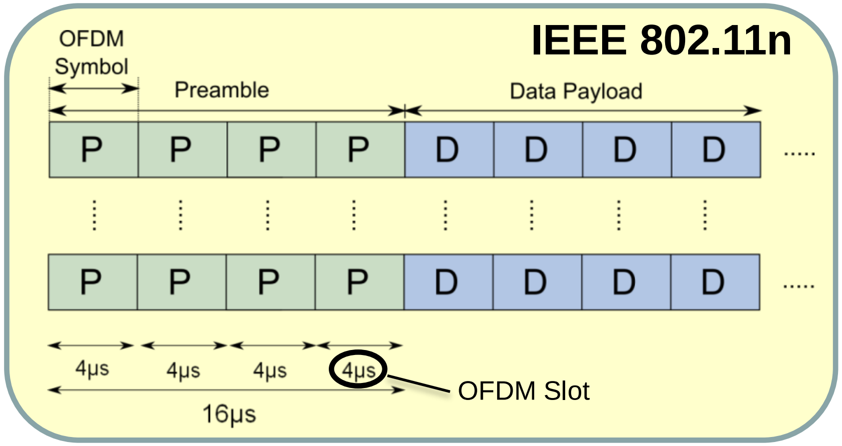
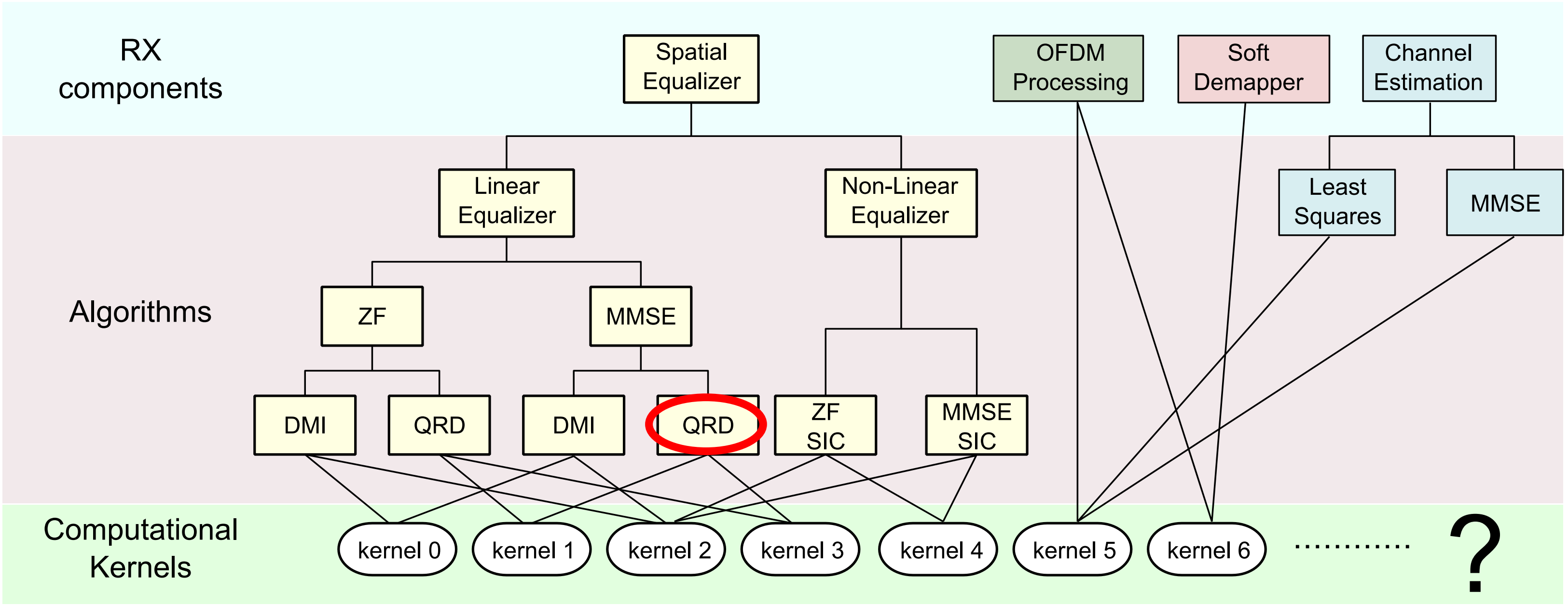
MIMO OFDM Application: Transceiver Structure



- **Outer Modem**
  - Channel (De-)coding
  - (De-)Interleaving
- **Inner Modem** (RX)
  - RX OFDM Processing
  - Channel Estimation
  - Spatial Equalizing: Mitigate channel impact on payload
  - Soft Demapping: Calculate soft bits (LLRs)
    BPSK, 4QAM, 16QAM, 64QAM



MIMO OFDM Application: Kernel Identification



- **Analyze different algorithmic choices within RX blocks**
  - Identify computational kernels
    - Recurring tasks
    - Operate on data with certain alignment
- **Build application as composition of kernels**

- **LMMSE MIMO Equalizer** with QRD
  - Basic transmission equation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

  - Linear MMSE equalization

$$\hat{\mathbf{x}} = \mathbf{G}\mathbf{y}, \quad \mathbf{G} = \left(\hat{\mathbf{H}}^{\mathbf{H}}\hat{\mathbf{H}} + \frac{\sigma_n^2}{E_s}\mathbf{I}\right)^{-1}\hat{\mathbf{H}}^{\mathbf{H}}$$

  - Regularized QRD

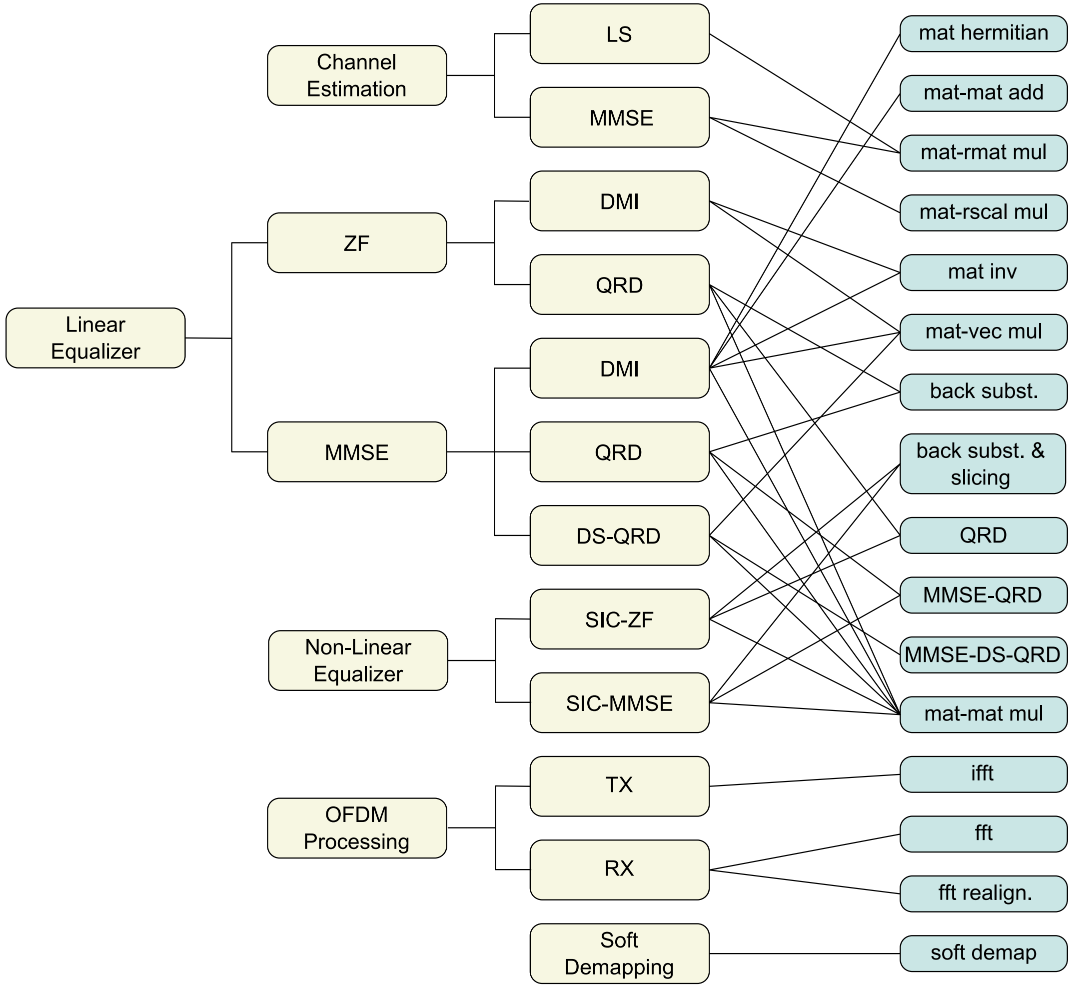
$$\overline{\mathbf{H}} = \begin{pmatrix} \hat{\mathbf{H}} \\ \frac{\sigma_n}{\sqrt{E_s}}\mathbf{I} \end{pmatrix} = \begin{pmatrix} \mathbf{Q}_{\mathbf{a}} \\ \mathbf{Q}_{\mathbf{b}} \end{pmatrix}\mathbf{R}$$

  - Rewrite **G** using $\mathbf{Q}_a$ and $\mathbf{Q}_b$

$$\mathbf{G} = \frac{\sqrt{E_s}}{\sigma_n}\mathbf{Q}_{\mathbf{b}}\mathbf{Q}_{\mathbf{a}}^{\mathbf{H}}$$

- **Computational Kernels**
  - Regularized QR decomposition
  - Matrix-matrix multiplication
  - Matrix-vector multiplication

MIMO OFDM Application: Kernel Overview



- **Application variants consist of a few kernels only**
- **Kernels implement vector arithmetic**
- **Suitable platform hast to exploit data level parallelism (DLP)**

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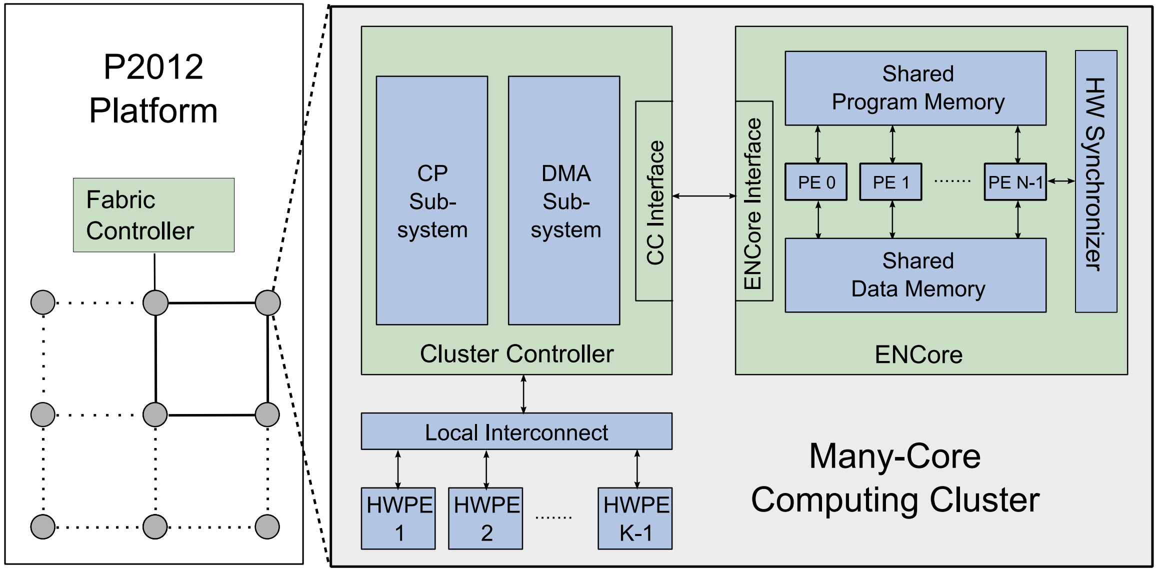
- **Two common approaches to exploit DLP**
  - Very Long Instruction Word (VLIW) architectures
    - Instructions are packed into macro instruction and executed in parallel
    - Example: TI TMS320C6000
  - Single Instruction Multiple Data (SIMD) architectures
    - One instruction is executed on a set of data
    - Example
      - ST Ericsson EVP
      - Freescale MSC8156
      - STM P2012
  - Regular data accesses and vectorial kernels call for SIMD architecture

- **SoC platform with maximum of 32 clusters**
- **One cluster provides**
  - Max. 16 RISC cores (STxP70) @ 600MHz
  - VECx vector extension (SIMD)
    - 128 bit vector registers
    - **8x16 bit** or 4x32 bit operations
  - Hardware synchronizer for inter-core signaling
  - Interface for hardware accelerators (ASICs)



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- **Problem**
  - Strict real time constraints of standards imply use of fixed-point operations
  - ASIC implementations choose fixed-point bitwidth freely
  - DSPs traditionally use 16bit data types
  - **Challenge for numerical stability!**

- **Critical point**
  - Matrix Inversions
  - Values run out of fixed point range
  - Example: MIMO Preprocessing

$$\mathbf{G} = \arg\min_{\mathbf{G}} E\left\{ \left| \mathbf{x} - \mathbf{G}\mathbf{y} \right|^2 \right\} = \left(\hat{\mathbf{H}}^H \hat{\mathbf{H}} + N_0 \mathbf{I} \right)^{-1} \hat{\mathbf{H}}^H$$

- **QR Decomposition of augmented channel matrix**

$$\overline{\mathbf{H}} = \begin{pmatrix} \hat{\mathbf{H}} \\ N_0 \mathbf{I} \end{pmatrix} = \mathbf{QR} = \begin{pmatrix} \mathbf{Q_a} \\ \mathbf{Q_b} \end{pmatrix} \mathbf{R} \quad \mathbf{Q}^H \mathbf{Q} = \mathbf{I} \quad \mathbf{Q}_a \mathbf{R} = \mathbf{H}$$

- **Rewriting equalizer matrix**

$$\mathbf{G} = N_0 \mathbf{Q_b} \mathbf{Q_a^H}$$

- **Choosing Modified Gram-Schmidt (MGS) as QRD algorithm**
  - Delivers $\mathbf{Q_b}$ for calculation of $\mathbf{G}$
  - Project and subtract column vectors for linear independence

for $j = 0$ to $N_t - 1$ **do**
  for $i = 0$ to $j - 1$ **do**
    $r_{ji} \leftarrow \mathbf{v_j^H v_i}$
    $\mathbf{v_j} \leftarrow \mathbf{v_j} - \mathrm{proj}_{\mathbf{v_i}}(\mathbf{v_j}) = \mathbf{v_j} - r_{ji}\mathbf{v_i}$
  end for
  $r_{jj} \leftarrow \|\mathbf{v_j}\|$
  $\mathbf{v_j} \leftarrow \frac{\mathbf{v_j}}{r_{jj}}$
end for
$Q \leftarrow [\mathbf{v_0}, \mathbf{v_1}, ..., \mathbf{v_{N_t-1}}]$

- **Problem**
  - Repeated projection and subtraction may cause values to run out of fixed point range
  - Problem increases with number of spatial streams (4x4)
- **Mitigation: Dynamic Scaling**
  - One column vector is projected and subtracted from right hand vectors
  - Check whether vectors exceed certain range and shift back

Algorithm 1 MMSE MGS-QRD with DS

$\mathbf{V} \leftarrow \bar{\mathbf{H}}$
for $i = 1$ to N_t **do**
  for $j = i$ to N_t **do**
    if $\max\{|\Re\{v_{j,1}\}|, |\Im\{v_{j,1}\}|, ..\} < B_l$ **then**
      $\mathbf{v_j} \leftarrow 2\mathbf{v_j}$
    else if $\max\{|\Re\{v_{j,1}\}|, |\Im\{v_{j,1}\}|, ..\} > B_h$ **then**
      $\mathbf{v_j} \leftarrow \mathbf{v_j}/2$
    end if
  end for
  $\mathbf{v_i} \leftarrow \mathbf{v_i}/\|\mathbf{v_i}\|$
  for $j = i + 1$ to N_t **do**
    $\mathbf{v_j} \leftarrow \mathbf{v_j} - \left(\mathbf{v_i}^H \mathbf{v_j}\right) \mathbf{v_i}$
  end for
end for
$\mathbf{Q} \leftarrow [\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_{N_t}}]$

Dynamic Scaling (lines 3–9)

Fixed-Point Aspects: Mitigation 3

- **Problem**
  - In high SNR region, scaled identity matrix in augmented channel matrix becomes too small to calculate reliant Qb

$$\overline{\mathbf{H}} = \begin{pmatrix} \hat{\mathbf{H}} \\ N_0 \mathbf{I} \end{pmatrix} = \mathbf{QR} = \begin{pmatrix} \mathbf{Q_a} \\ \mathbf{Q_b} \end{pmatrix} \mathbf{R}$$

- **Mitigation**
  - Unified Regularized Channel Matrix (URCM)
  - Scale up identity matrix

$$\overline{\mathbf{H}}_u = \begin{pmatrix} \hat{\mathbf{H}} \\ \mathbf{I} \end{pmatrix}$$

  - Correction factor in projection
  - No adaption in subtraction

Algorithm 2 MMSE MGS-QRD with DS and URCM

$\mathbf{V} \leftarrow \bar{\mathbf{H}}_\mathbf{u}$
for $i = 1$ to N_t **do**
    $\xi_i = (\mathbf{H}^H\mathbf{H})_{i,i} + N_0$
end for
for $i = 1$ to N_t **do**
    for $j = i$ to N_t **do**
        if $\max\{|\Re\{v_{j,1}\}|, |\Im\{v_{j,1}\}|, ..\} < B_l$ **then**
            $\mathbf{v_j} \leftarrow 2\mathbf{v_j}$
            $\xi_j \leftarrow 4 \cdot \xi_j$
        else if $\max\{|\Re\{v_{j,1}\}|, |\Im\{v_{j,1}\}|, ..\} > B_h$ **then**
            $\mathbf{v_j} \leftarrow \mathbf{v_j}/2$
            $\xi_j \leftarrow 1/4 \cdot \xi_j$
        end if
    end for
    $\mathbf{v_i} \leftarrow \mathbf{v_i}/|\sqrt{\xi_i}|$
    for $j = i+1$ to N_t **do**
        $s = \left(\mathbf{v_i}^H \odot \mathbf{a}^T\right)(\mathbf{v_j} \odot \mathbf{a})$
        $\mathbf{v_j} \leftarrow \mathbf{v_j} - s\mathbf{v_i}$
        $\xi_j \leftarrow \xi_j - |s|^2$
    end for
end for
$\mathbf{Q} \leftarrow [\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_{N_t}}]$

- **Status**
  - Current algorithm allows 4x4 MIMO LMMSE Detection with algorithmic performance close to floating point
- **Limitation**
  - Matrix R is lost due to DS
  - No Sorting
  - Both expected certain other MIMO detector types
    - MMSE-SIC
    - Sphere Detection
- **Mitigation**
  - Keep track of DS shifts to restore R and original column norms

Algorithm 3 MMSE MGS-SQRD with DS and URCM

$\mathbf{V} \leftarrow \bar{\mathbf{H}}_\mathbf{u}$
$\mathbf{P} \leftarrow \mathbf{I}_{N_t}$
for $i = 1$ to N_t **do**
  $\xi_i = \left(\mathbf{H}^H\mathbf{H}\right)_{i,i} + N_0$
  $e_i = 0$
end for
for $i = 1$ to N_t **do**
  $k = \text{argmin}_{j=i,\ldots,N_t}(\xi_j)$
  exchange columns i and k in $\mathbf{V}$, $\mathbf{R}$ and $\mathbf{P}$
  exchange elements i and k in ξ and $\mathbf{e}$
  for $j = i$ to N_t **do**
    if $\max\{|\Re\{v_{j,1}\}|, |\Im\{v_{j,1}\}|, ..\} < B_l$ **then**
      $\mathbf{v_j} \leftarrow 2\mathbf{v_j}$
      $e_j \leftarrow e_j + 1$
      $\xi_j \leftarrow 4 \cdot \xi_j$
    else if $\max\{|\Re\{v_{j,1}\}|, |\Im\{v_{j,1}\}|, ..\} > B_h$ **then**
      $\mathbf{v_j} \leftarrow \mathbf{v_j}/2$
      $e_j \leftarrow e_j - 1$
      $\xi_j \leftarrow 1/4 \cdot \xi_j$
    end if
  end for
  $r_{ii} = 1/\sqrt{\xi_i} \cdot 2^{+e_i}$
  $\mathbf{v_i} \leftarrow \mathbf{v_i}/\sqrt{\xi_i}$
  for $j = i + 1$ to N_t **do**
    $s = \left(\mathbf{v_i}^H \odot \mathbf{a}^T\right)\left(\mathbf{v_j} \odot \mathbf{a}\right)$
    $r_{ij} = s \cdot 2^{-e_j}$
    $\mathbf{v_j} \leftarrow \mathbf{v_j} - s\mathbf{v_i}$
    $\xi_j \leftarrow \xi_j - |s|^2$
  end for
end for
$\mathbf{Q} \leftarrow [\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_{N_t}}]$

Sorting

Dynamic Scaling

Project & Subtract

Tracking shifts and column norms

URCM

Restore R matrix

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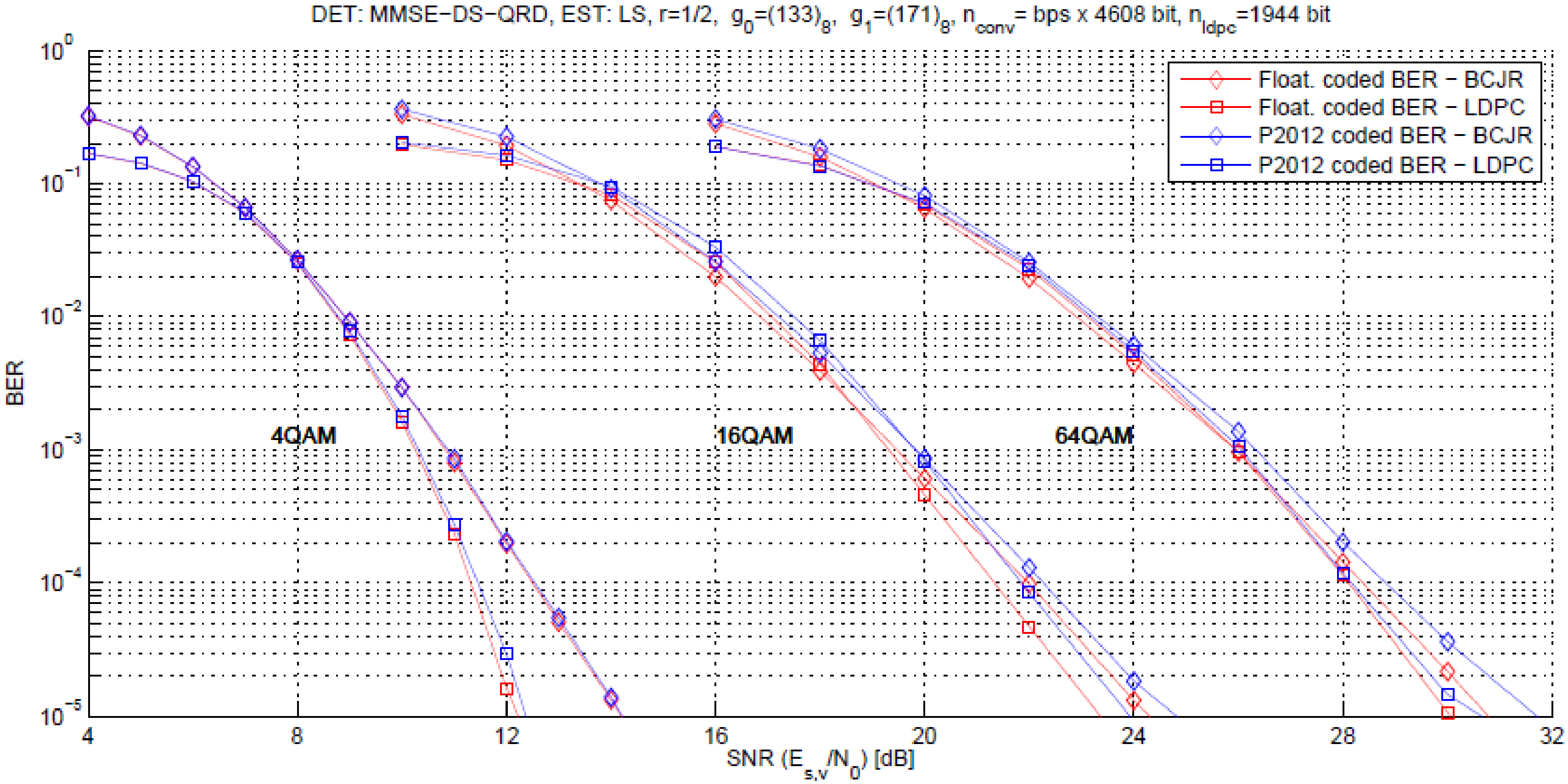
- **Channel Simulation**
  - AWGN
  - Rayleigh Fading (20dB drop along 150ns)



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- **Algorithmic improvements (DS, URCM) come at the cost of increasing execution time**

System	2x2		4x4	
Operation	cycles	T (μs)	cycles	T (μs)
MIMO Preprocessing (per frame)				
mgs-mmse-qrd	22,848	38.08	55,536	92.56
mgs-mmse-ds-qrd (UMCR)	35,424	59.04	66,624	111.04
mgs-mmse-ds-sqrd (UMCR)	43,248	72.08	85,392	142.32
gr-mmse-sqrd	-	-	112,032	186.72
matrix-matrix mul.	2,496	4.16	11,472	19.12
sinr-calc-r	9,456	15.76	25,824	43.04
sinr-calc-r-inv	7,248	12.08	15,600	26.00
Spatial Equalizing (per OFDM slot)				
back substitution	1,188	1.98	2,736	4.56
matrix-vector mul.	1,968	3.28	3,312	5.52

- **Note**
  - QRD algorithms with lower operation count (Givens Rotation) are not faster on SIMD platform
  - Reason: Irregular data accesses

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Summary & Outlook

- **Summary**
  - Numerical stability is a critical point in MIMO detection
  - MIMO detection can reach close to floating point algorithmic performance on 16bit fixed point DSPs
  - Moderate additional costs in execution time

- **Outlook**
  - VLIW architectures
  - Advanced, iterative receivers
  - Customized ASIP for baseband processing

Thank you!