Joint Optimal Power Allocation and Relay Selection with Spatial Diversity in Wireless Relay Networks

Md Habibul Islam\textsuperscript{1}, Zbigniew Dziong\textsuperscript{1}, Kazem Sohraby\textsuperscript{2}, Mahmoud F Daneshmand\textsuperscript{3}, and Rittwik Jana\textsuperscript{4}

\textsuperscript{1}École de Technologie Supérieure (ETS), University of Quebec, Montreal, Quebec H5A 1K6, Canada
\hspace{1em} e-mail: md-habibul.islam.1@ens.etsmtl.ca, zbigniew.dziong@etsmtl.ca
\textsuperscript{2}Department of Electrical Engineering, University of Arkansas, 3217 Bell Engineering Center Fayetteville, AR 72701
\hspace{1em} e-mail: sohraby@uark.edu
\textsuperscript{3}AT&T Labs Research, 200 S Laurel Ave - Bldg D, Middletown, NJ
\hspace{1em} e-mail: daneshmand@att.com
\textsuperscript{4}AT&T Labs Research, 180 Park Ave - Building 103, Florham Park, NJ
\hspace{1em} e-mail: rjana@research.att.com

Abstract—We consider a wireless relay network (WRN) where multiple mobile stations (MSs) try to send their data to a base station (BS) either directly or via a set of fixed relay stations (RSs). For this network, we study the problem of joint optimal MS and RS power allocation and relay selection with the objective of minimizing the total transmitted power of the system. The joint optimization algorithm must satisfy the minimum data demand of each MS. We formulate the problem as a mixed integer nonlinear programming (MINLP) problem and find the solution under different relaying architectures and spatial diversity schemes. The optimal solution of the MINLP problem is exponentially complex due to its combinatorial nature. We use the MATLAB based commercial software TOMLAB to find a near optimal solution of the original problem by applying a simple relay selection scheme based on the channel gains between MSs and RSs. Numerical results are presented to show the performance of this simple scheme with respect to the near optimal solution in terms of total power consumption.

I. INTRODUCTION

The ever increasing demand for high data rate services has resulted in a significant amount of energy consumption by the communication component of information and communication technology (ICT). As a consequence, the ICT is playing a major role in global climate change that demands substantial reduction in world-wide energy consumption. Finding alternative ways to improve energy efficiency and thus reducing the energy consumption of wireless networks is vital for a greener future.

Given the obvious need to reduce the energy consumption, the fundamental challenge is how to reduce the overall power consumption of wireless networks while maintaining adequate coverage, quality of services, and reliability. Wireless relay networks (WRNs) can provide a favorable platform to address this challenge. The underlying technology of WRNs is cooperative communications, which is shown to be a promising approach to increase data rates and reliability in wireless networks [11]–[3]. In WRNs, lower energy consumption is achieved via using less transmission power due to smaller distances between relays and the terminals, spatial diversity, and using efficient signal processing schemes such as distributed beamforming [4], distributed space-time coding [5], [6], etc. On the other hand, power control is recognized as a powerful tool to minimize total transmission power of wireless communications systems. In a wireless relay network (WRN), the choice of relay stations (RSs) to be optimally assigned to the mobile stations (MSs) is critical to the overall network performance.

It has been observed that regardless of the relaying schemes applied, e.g. amplify-and-forward (AF) and decode-and-forward (DF), the performance of cooperative communications highly depends on the efficient selection of relays for the sources and the power control across the transmissions [7]. Joint power allocation and relay selection in multi-user scenarios have been studied in [7]–[10]. In [8], in order to maximize the system capacity with low computational complexity and system overhead, the authors propose to design effective relaying algorithms by jointly optimizing relay node selection and power allocation for AF wireless relay networks with multiple sources and a single destination. In [9], the authors consider joint optimization of power allocation and relay selection for AF relay networks with multiple source-destination pairs. The joint schemes are proposed under two types of design criteria: i) maximization of user rates, and ii) minimization of the total transmit power at the relays. Unlike the above works, the authors in [10] develop a strategy to minimize the total transmit power in a DF user cooperative uplink, such that each user satisfies its required data rate. In [10], the authors model the total power minimization problem as an optimization problem where the objective function (total network power) is a convex function of user powers and the constraints are target rates of users which are concave functions. They then solve the optimization problem by Lagrange multiplier method. A common assumption in all these works [8]–[10] is that the transmissions from sources are orthogonal to each other, i.e. the channel is not interference-limited. For interference-limited DF WRN with multiple source-destination pairs and a pool of available relays, Gkatzikis and Kout-
N806.16j type networks. Let the number of MSs and RSs be such that a deployment scenario is more representative of IEEE deployed and the locations of RSs are fixed. It should noted of the service area. Within the same area, multiple RSs are also MSs are uniformly distributed. A BS is deployed at the center of system scenarios.

II. System Model

We consider a hexagonal service area, where a number of MSs are uniformly distributed. A BS is deployed at the center of the service area. Within the same area, multiple RSs are also deployed and the locations of RSs are fixed. It should noted that such a deployment scenario is more representative of IEEE 806.16j type networks. Let the number of MSs and RSs be \( N_{\text{MS}} \) and \( N_{\text{RS}} \), respectively. An MS can either be directly connected to the BS or via one or more RSs. We assume that if an MS is directly connected to the BS, it cannot be connected to an RS, and vice versa. However, both BS and RSs can be accessed directly connected to the BS, it cannot be connected to an RS, or via one or more RSs. We assume that if an MS is directly connected with a BS or it is assisted by relays to transmit its data to BS.

\[
x_k = \begin{cases} 
0 & \text{if } \text{MS}_k \text{ is directly connected with BS} \\
1 & \text{otherwise.}
\end{cases}
\]

\[
y_{km} = \begin{cases} 
1 & \text{if MS}_k \text{ is connected with RS}_m, \\
0 & \text{otherwise.}
\end{cases}
\]

If an MS is directly connected with the BS, it needs only one time slot to transmit its data to BS. On the other hand, if an MS is connected to the BS via an RS, then in the first time slot, an MS transmits unit energy signal to an RS. In the subsequent time slot, assuming the BS knows the channel state information (CSI) for the MS-RS link, the BS normalizes the received signal and retransmits to the destination BS.

III. Problem Formulation

In this work, we want to solve the following joint optimization problem. Given the location of the BS and a set of fixed RSs, find the optimal power allocations \( \{P_k\} \), \( \{Q_{km}\} \), and \( \{F_{km}\} \), and optimal selection variables \( \{x_k\} \), \( \{y_{km}\} \) such that the total transmit power of the system is minimized while the minimum data rate demand \( \{r_{\text{min}}^k\} \) of each MS is met.

Using the notations defined in the previous section, the above optimization problem can be mathematically expressed as follows.

\[
\begin{align*}
\min & \sum_{k=1}^{N_{\text{MS}}} P_k + \sum_{k=1}^{N_{\text{MS}}} \sum_{m=1}^{N_{\text{RS}}} Q_{km} + \sum_{k=1}^{N_{\text{MS}}} \sum_{m=1}^{N_{\text{RS}}} F_{km} \\
\text{s.t.} & \quad r_k \geq r_{\text{min}}^k, \quad \forall k \\
& \quad x_k + \sum_{m=1}^{N_{\text{RS}}} y_{km} = R, \quad \forall k \\
& \quad x_k y_{km} = 0, \quad \forall k, m \\
& \quad 0 \leq P_k \leq P_{\text{max}}^k x_k, \quad \forall k \\
& \quad 0 \leq Q_{km} \leq Q_{\text{max}} y_{km}, \quad \forall k, m \\
& \quad 0 \leq F_{km} \leq F_{\text{max}} y_{km}, \quad \forall k, m \\
& \quad 0 \leq \sum_{k=1}^{N_{\text{MS}}} F_{km} \leq F_{\text{max}}, \quad \forall m \\
& \quad x_k \in \{0, 1\}, \quad y_{km} \in \{0, 1\}, \quad \forall k, m
\end{align*}
\]

variables: \( \{x_k\} \), \( \{y_{km}\} \), \( \{P_k\} \), \( \{Q_{km}\} \), \( \{F_{km}\} \)

The objective function (1a) minimizes the total transmitted power of the system. Constraint (1b) ensures that the data channel gains from MS\(_k\) to BS, from MS\(_k\) to RS\(_m\), and from RS\(_m\) to BS are captured by the parameters \( g_k, h_{km}, \) and \( d_m \), respectively. All the channel gains may include the effect of path loss, shadowing, and fading. Let \( P_k \) denote the power transmitted by MS\(_k\) if MS\(_k\) is directly connected with the BS. Let \( Q_{km} \) and \( F_{km} \) be the powers transmitted by MS\(_k\) and RS\(_m\), respectively, in the links MS\(_k\)-RS\(_m\)-BS if MS\(_k\) is connected to BS via RS\(_m\). The maximum transmit power budget constraint of an MS and an RS are \( P_{\text{max}} \) and \( F_{\text{max}} \), respectively. The variances of additive white Gaussian noise (AWGN) at BS and RS are denoted by \( \sigma^2_0 \) and \( \sigma^2_2 \), respectively.
transmission rate of each MS is larger than its minimum rate requirements. In (1b), $r_k$ is the maximum achievable transmission rate of MS$_k$. In the next section, we present the expressions for $r_k$ under different spatial diversity schemes. Constraint (1c) along with the constraint (1d) states that an MS is either directly connected with the BS or via a single or multiple RSs, and if an MS is directly connected with the BS, it cannot be assigned with one or multiple RSs and vice versa. In (1e), $R$ is a predetermined system parameter which represents the exact number of RSs assigned with each MS if the MS is not directly connected with the BS. Based on the relaying architecture and spatial diversity schemes, in this work, we set $R = 1$ or $R = 2$. The non-negativity of the power allocation variables as well as the power budget constraints of MSs and RSs are ensured by constraints (1e), (1f), (1g), and (1h). The conditions that if $x_k = 0$, $P_k = 0$, if $y_{km} = 0$, $Q_{km} = 0$, and if $g_{km} = 0$, $F_{km} = 0$, are also captured in the constraints (1e), (1f), and (1g). Finally, constraint (1i) satisfies the condition that MS-BS and MS-RS assignment variables are binary.

IV. TRANSMISSION RATES UNDER DIFFERENT SCHEMES

In this paper, we solve the total transmit power minimization problem (1) under different deployment scenarios and spatial diversity schemes. Specifically, we consider the following scenarios:

- **BS-only architecture:** Under this architecture, the network consists of BS and MSs only. Since there are no relays, all MSs transmit directly to BS and $x_k = 1$, $\forall k \in [1, N_{ms}]$. We consider this scenario to show the advantage of using relays over non-relay networks in terms of energy saving.

- **Single relay per MS:** Under this scheme, if an MS is not directly connected with a BS, it would transmit to a BS via exactly a single RS. Therefore, under this scheme, $R = 1$. In this scheme, spatial diversity is achieved through AF relaying scheme.

- **Multiple relays per MS:** Unlike the scenario of single relay per mobile, in this scenario, if an MS is not directly connected to a BS, it would transmit to a BS via $R > 1$ relays. For the sake of simplicity, here, we consider $R = 2$. Under this scheme, additional spatial diversity is achieved due to the multipath combining of the received signal from multiple relays at the BS.

- **Distributed beamforming:** In this case, a maximum ratio transmission (MRT) based distributed beamforming scheme would be employed by multiple RSs to assist MSs to transmit their data to BS.

- **Distributed space-time coding:** In this scheme, multiple RSs would employ the distributed space-time coding [5], [6] to assist MSs which are not directly connected to the BS.

Under the BS-only non-relay deployment scenario, the maximum achievable data rate of MS$_k$, $\forall k \in [1, N_{ms}]$, can be expressed by the well-known Shannon capacity theorem:

$$r_k = W \log_2 \left( 1 + \frac{x_k P_k |g_{km}|^2}{\sigma_0^2} \right),$$  

(2)

where $W$ is the bandwidth of the channel. Without the loss of generality, we can assume $W = 1$.

Now, we look at the case when the MSs which cannot directly transmit to BS are assisted by exactly $R$ number of relays. In this case, under AF scheme [1], [9], the data rate of MS$_k$ is given by

$$r_k = \log_2 \left( 1 + \frac{x_k P_k |g_{km}|^2}{\sigma_0^2} \right) + \log_2 \left( 1 + \frac{N_{ms}}{R} \sum_{m=1}^{N_{ms}} \gamma_{km} \right),$$  

(3)

where

$$\gamma_{km} = \frac{y_{km} Q_{km} |h_{km}|^2 F_{km} |d_m|^2}{y_{km} F_{km} |d_m|^2 \sigma_r^2 + \left( y_{km} Q_{km} |h_{km}|^2 + \sigma_0^2 \right) \sigma_r^2},$$  

(4)

Note that in (3), $R = 1$ represents the scenario where each MS without any direct connection with BS is assisted by one RS, and $R = 2$ represents the scenario where each MS without any direct connection with the BS is assisted by two RSs.

A. Distributed Beamforming

For the distributed beamforming case, if an MS is not directly connected with the BS, it is assisted by $R$ number of relays. For the sake of simplicity, we limit this to $R = 2$. Given the coordinates of the locations of RSs, for each RS, we select the closest RS as its pair. As a consequence, an RS might appear in a single or multiple RS pairs. Note that the above method of choosing RS pairs is not necessarily optimal. Let $N_{rs}$ be the total number of RS pairs. Denote $RSP_i, \forall i \in [1, N_{rs}]$, as the $i$th RS pair. To this end, we define the following binary assignment variable:

$$\alpha_{kl} = \begin{cases} 1 & \text{if MS}_k \text{ is assisted by } RSP_i \\ 0 & \text{otherwise} \end{cases}$$

With a little abuse of notations, we denote the vectors of complex channel gains from MS$_k$ to RSP$_{i1}$ and from RSP$_{i2}$ to the BS as $h_{kl} = [h_{kl}^{(1)}, h_{kl}^{(2)}]$ and $d_i = [d_i^{(1)}, d_i^{(2)}]^T$, respectively. In AF distributed beamforming, during the first time slot, MS$_k, \forall k \in [1, K]$ transmits signal to RSP$_i, \forall i \in [1, 2]$ of RSP$_i$ using the transmit power $Q_{kl}^{(i)}$. In the second time slot, each RS$_i$ of RSP$_i$ normalizes the received signal, multiplies it by a beamforming coefficient and transmit the amplified signal to the BS using its transmit power $F_{kl}^{(i)}$. Let $w_{kl} = [w_{kl}^{(1)}, w_{kl}^{(2)}]^T$ be the beamforming weight vector employed by RSP$_{i}$ to transmit the signal of MS$_k$ to BS. Now, with AF distributed beamforming scheme, the data rate of MS$_k$, $\forall k \in [1, N_{ms}]$, can be written as

$$r_k^{rs} = \log_2 \left( 1 + \frac{x_k P_k |g_{km}|^2}{\sigma_0^2} \right) + \sum_{i=1}^{N_{rs}} r_{kl},$$  

(5)
where
\[ r_{kl} = \log_2 \left( 1 + \frac{1}{2} \sum_{i=1}^{2} \gamma_{kl}^{(i)} \right), \]  
and
\[ \gamma_{kl}^{(i)} = \frac{\alpha_k Q_{kl}^2 |F_{kl}^{(i)}| |h_{kl}^{(i)}|^2 |d_{i}^{(i)}|^2 |w_{kl}^{(i)}|^2}{\alpha_k F_{kl}^2 |d_{i}^{(i)}|^2 |w_{kl}^{(i)}|^2 \sigma_0^2 + (Q_{kl}^2 |h_{kl}^{(i)}|^2 + \sigma_0^2) \sigma_0^2}. \]  

In this paper, we employ the simple maximum ratio transmission (MRT) beamforming scheme under which the beamforming vector \( w_{kl} \) can be expressed as
\[ w_{kl} = \frac{f_{kl}^{(i)}}{||f_{kl}||} \quad \forall k \in [1, n_{ms}], \forall l \in [1, n_{raw}], \]  
where \( f_{kl} = [h_{kl}^{(1)} d_{1}^{(i)}, h_{kl}^{(2)} d_{2}^{(i)}]^T \) is the equivalent channel gain vector for the MS\( k \)-RS\( l \) link.

It should be noted that MRT based distributed beamforming scheme requires a centralized control with access to all channel information. We assume that BS has perfect knowledge of all channel information and it feeds back those information to RSs.

### B. Distributed Space-Time Coding

In distributed space-time coding, if MS\( k \) wants to send the signal \( s_k = [s_k^{(1)}, \ldots, s_k^{(T)}]^T \) in the codebook \( \{s_k^{(1)}, \ldots, s_k^{(L)}\} \) to BS via RSP\( l \), where \( T \) is the length of the time slot, then the received signal at RS\( l \) of RSP\( l \), and at BS can be expressed, respectively, as [5], [6]
\[ r_{kl}^{(i)} = \sqrt{T h_{kl}^{(i)} s_k + u}, \]  
\[ x_{kl}^{(i)} = \frac{2}{\sum_{i=1}^{T} d_{i}^{(i)} t_{kl}^{(i)}} + \nu, \]  
where \( u \) and \( v \) are \( T \times 1 \) zero-mean complex AWGN vectors at RSs and BS, respectively with component wise variances \( \sigma_0^2 \) and \( \sigma_0^2 \) and
\[ t_{kl}^{(i)} = \sqrt{\frac{P_{kl}}{Q_{kl}^2 + \sigma_0^2}} A_{kl}^{(i)} k_{kl}^{(i)} , \]  
where \( T \times T \) dimensional matrix \( A_{kl}^{(i)} \) corresponds to the \( i \)th column of a proper \( T \times T \) space-time code. In [5], authors designed the distributed space-time codes such that \( A_{kl}^{(i)} \) is a unitary matrix.

With distributed space-time coding, the capacity of MS\( k \), \( \forall k \in [1, n_{ms}] \), can be written as [5], [6]
\[ r_k^w = \log_2 \left( 1 + \frac{x_k P_k |g_k|^2}{\sigma_0^2} + \sum_{l=1}^{n_{raw}} \rho_{kl} \right), \]  
where
\[ \rho_{kl} = \log_2 \left( 1 + \frac{2}{\sum_{i=1}^{T} \mu_{kl}^{(i)} |h_{kl}^{(i)} d_{i}^{(i)}|^2} \right). \]  

\[ \mu_{kl}^{(i)} = \frac{\alpha_k Q_{kl}^2 |F_{kl}^{(i)}| |h_{kl}^{(i)}|^2 |d_{i}^{(i)}|^2 |w_{kl}^{(i)}|^2}{\alpha_k F_{kl}^2 |d_{i}^{(i)}|^2 \sigma_0^2 + (Q_{kl}^2 |h_{kl}^{(i)}|^2 + \sigma_0^2) \sigma_0^2}. \]  
is the portion of the average symbol energy passing from the RS\( i \) of RSP\( l \) to noise power ratio.

### V. Solution Approach

The optimization problem described in (1) is a mixed integer nonlinear programming (MINLP) problem, which is NP-hard in general, due to the discrete nature of the BS and RS selection variables, and the continuous nature of the power allocation variables. The optimal solution of (1) can be obtained by exhaustive search algorithm which is computationally intractable due to its exponential complexity with respect to the number of MSs and RSs. Some commercial software packages, e.g. TOMLAB [11], which uses branch-and-bound algorithm, may provide near-optimal solutions. In this section, we provide a heuristic algorithm to get sub-optimal solutions of the MINLP problem. The heuristic algorithm is similar to the one-shot greedy algorithm proposed in [7]. Under this scheme, first transmit power of each MS and RS are set to \( P_{max} \) and \( F_{max} \), respectively. Then, each MS greedily selects either the BS or a single or multiple RSs (based on different scenarios presented in Section IV) such a way that its data rate is maximized. With \( x_k \) and \( y_{km} \) fixed, the optimization problem (1) is no more an MINLP problem and can be easily solved using simple non-linear programming (NLP) tools. However, since the selection of BS or RSs for each MS does not consider the channel in the second hop, the solution of the MS and RS power allocation would be sub-optimal.

### VI. Numerical Results

In this section, we provide some numerical results to compare the performance of different schemes. In our simulation model, we consider a hexagonal cell with radius 1 km. The BS is located at the center of the cell. Within the cell, there are \( N_{ms} = 15 \) relays with their position fixed. The number of MSs is varied as \( N_{ms} = 30, 35, 40, 45, 50, 55, 60 \), and they are uniformly distributed within the cell. The normalized coordinates for the positions of BS, 15 RSs, and 30 MSs are shown in Fig 1 for a particular snapshot. The path loss exponent is 4. It is assumed that all the receivers at RSs and BSs are subject to Additive White Gaussian Noise (AWGN) with zero mean and unit variance. Channel coefficients are generated as circularly symmetric AWGN with zero mean and unit variance. Minimum traffic demand of MSs are uniformly generated in \([0, 1]\). The results of the simulations are averaged over 1000 channel realization. The locations of BS and RSs, and required data rate of each MS are fixed over all channel realizations. However, the locations of MSs change from one channel realization to another channel realization.

In Fig 2, we show the minimum total transmit power of six different relaying architectures and schemes for different numbers of MS served. The results obtained by using TOMLAB [11] are denoted by (O), and the results obtained by
the greedy algorithm are denoted by (G). We use the legends ‘BS-only’, ‘SRMS’, ‘DRMS’, ‘DBF’, and ‘DSTC’ to refer to the scenarios of BS-only architecture, single relay per MS, dual relay per MS, distributed beamforming, and distributed space-time coding, respectively. As can be seen from Fig 2 that the BS-only architecture requires more transmitted power to serve all MSs than the other five architectures. This result is expected since the MSs far from BS require to use more power to send their data to BS and achieve the target data rate. The single relay per MS architecture using both optimal (TOMLAB) and greedy schemes performs better than the BS-only architecture. It is also observed DRMS scheme performs better than SRMS scheme due to the multipath diversity captured by the DRMS scheme. On the other hand, DBF scheme provides both spatial diversity and array gain and thus performs better than the DRMS scheme that only takes the benefits of multipath diversity. Finally, our results show that DSTC scheme outperforms all other schemes in terms of minimum total power requirement. It is more likely that for the special case of 2 distributed antennas, coding and diversity gain achieved by DSTC is higher than the diversity and array gain provided by the DBF scheme. Finally, for all relay deployment scenarios, as expected, the performance of the greedy algorithm is worse than that of the optimal algorithm.

VII. CONCLUSION

We have studied the joint optimal MS and RS power control and BS and RS assignment to each MS for the uplink of a WRN, where multiple MSs send their data to BS either directly or via a single or multiple RSs. With the objective of minimizing the total transmit power of the system with the constraints on minimum data rate of each MS, and maximum transmit power budget of MSs and RSs, we have formulated the problem as an MINLP problem and then solved it under different system scenarios. The near-optimal solution of the MINLP problem has been obtained by the commercial software TOMLAB [11]. We have provided a heuristic solution based on a greedy approach and compared its performance with that of the near-optimal solution. Numerical results show that a gain of around 5 - 7 dB, in terms of total transmit power, can be achieved by exploiting spatial diversity inherent to the relaying architectures compared to the BS-only architecture.

REFERENCES