Design and Application of a Hilbert Transformer in a Digital Receiver

SDR'11 - WInnComm

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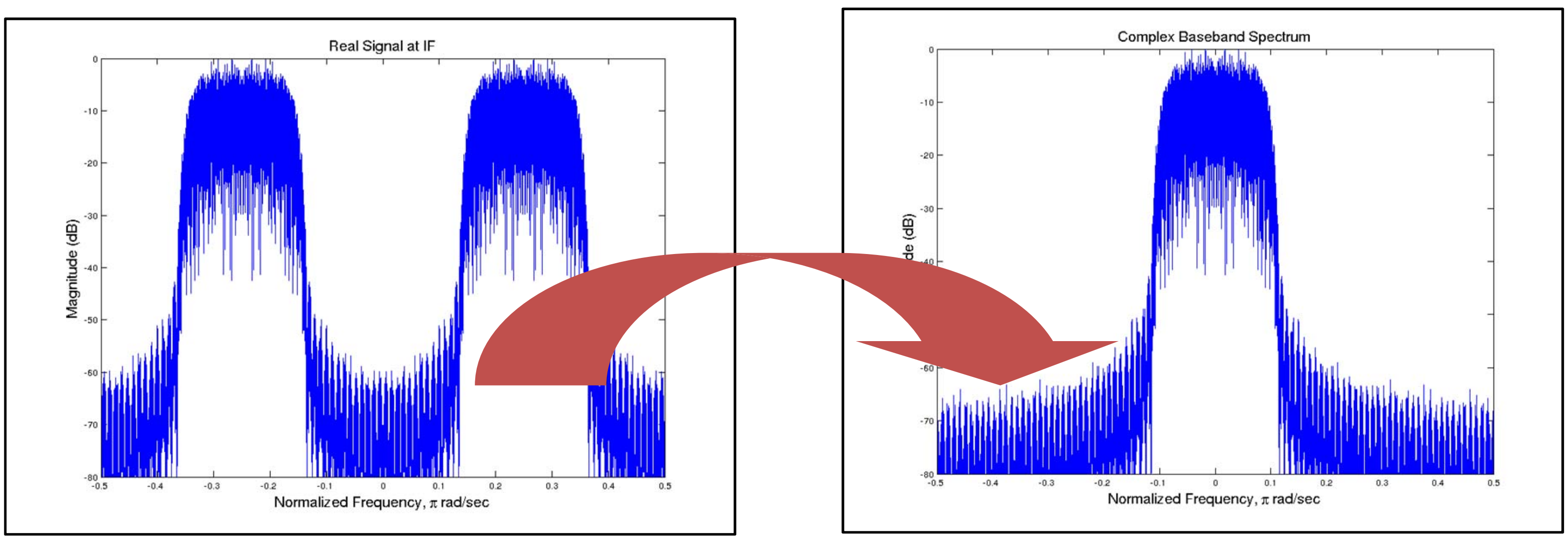
Motivation

- Given real spectrum at arbitrary IF, how to get to complex baseband?
- Constraints:
  - Real A/D
  - Minimize Processing Power

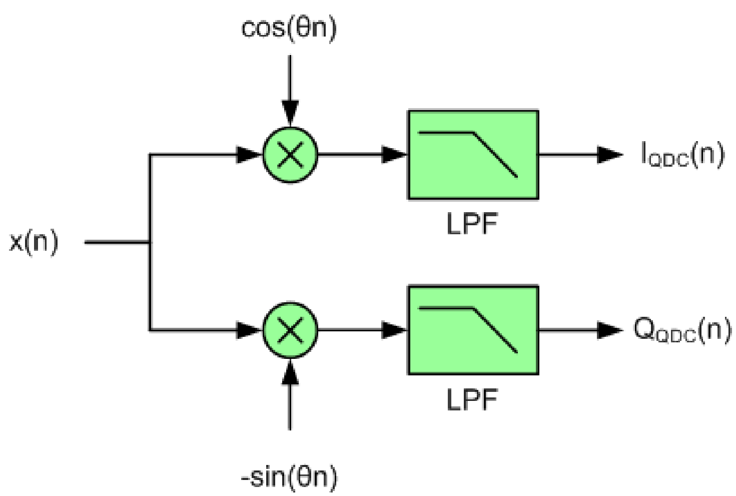


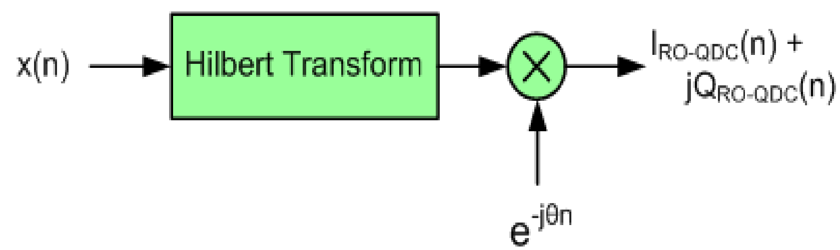
Outline

- ***Comparison of quadrature downconverter, downconversion with Hilbert transformer***
- Hilbert Transform Review
- Hilbert Transform Filter Design Through Windowing
- Hilbert Transform Filter Design in Frequency
- Designing a Half Band Filter
- Results
- Implementation of Hilbert Transform Filter

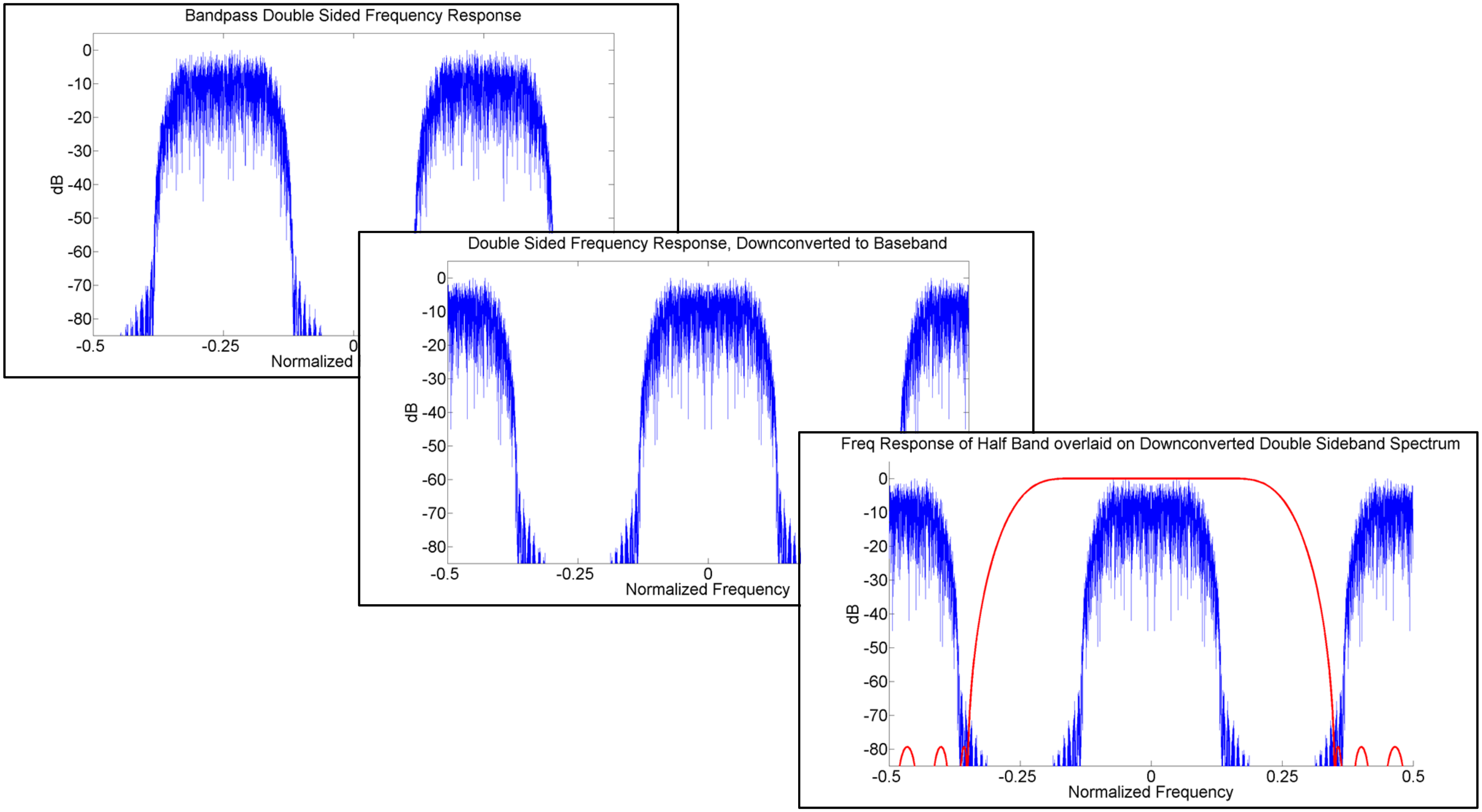
Downconversion Options

- Quadrature Downconverter
- Hilbert Transform + Heterodyne
- Other Options
  - Alias to baseband, Polyphase filter bank + FFT

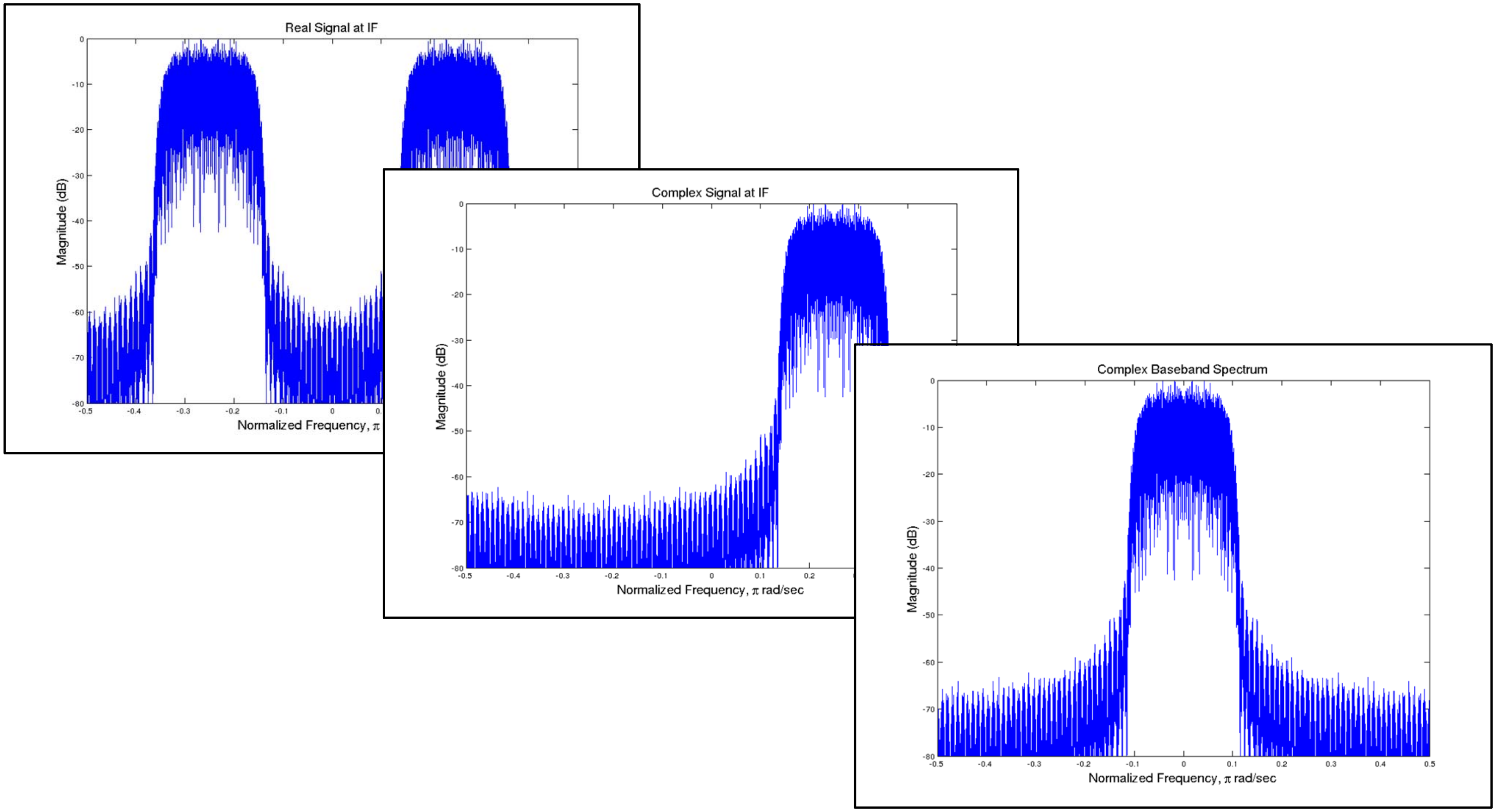




Downconversion With Quadrature Downconverter

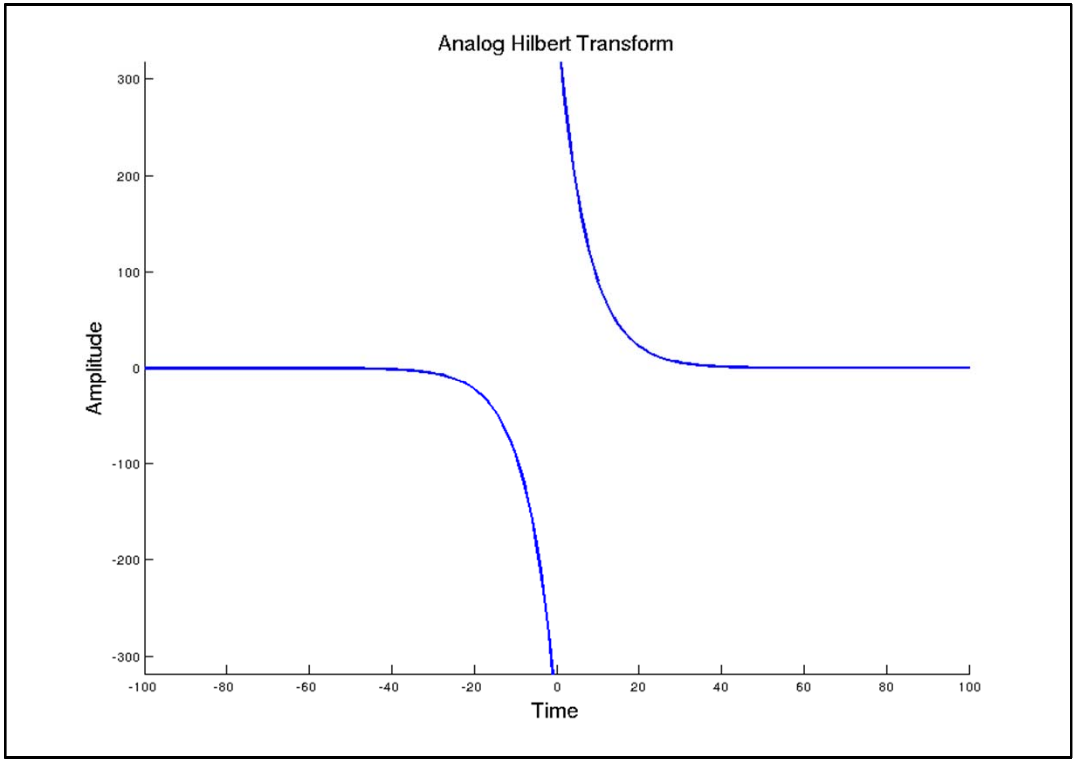


Downconversion With Hilbert Transform



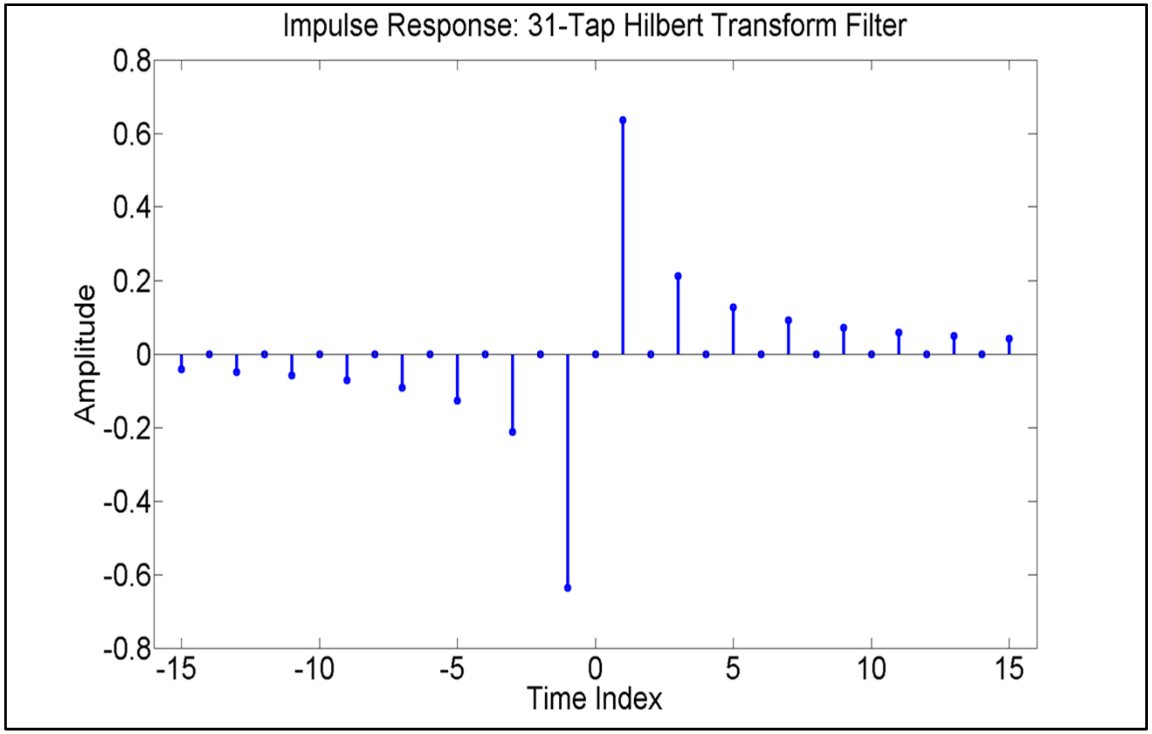
Hilbert Transform Review

- Convolutional Operator, Analog Representation
  - $x'(t) = x(t) * h(t)$
  - $h(t) = 1/\pi t$



Hilbert Transform Review (Con't)

- Digital Representation
  - x’[n] = x[n] * h[n]
  - h[n] = 2/(πn) for n odd
  - h[n] = 0 for n even
- Hilbert Transform
- Hilbert Transform***er***



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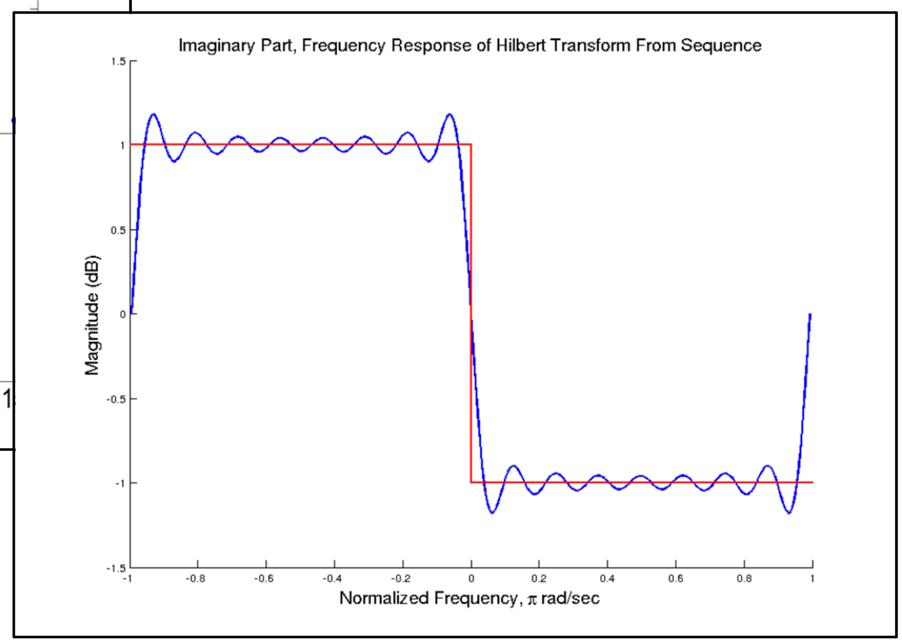
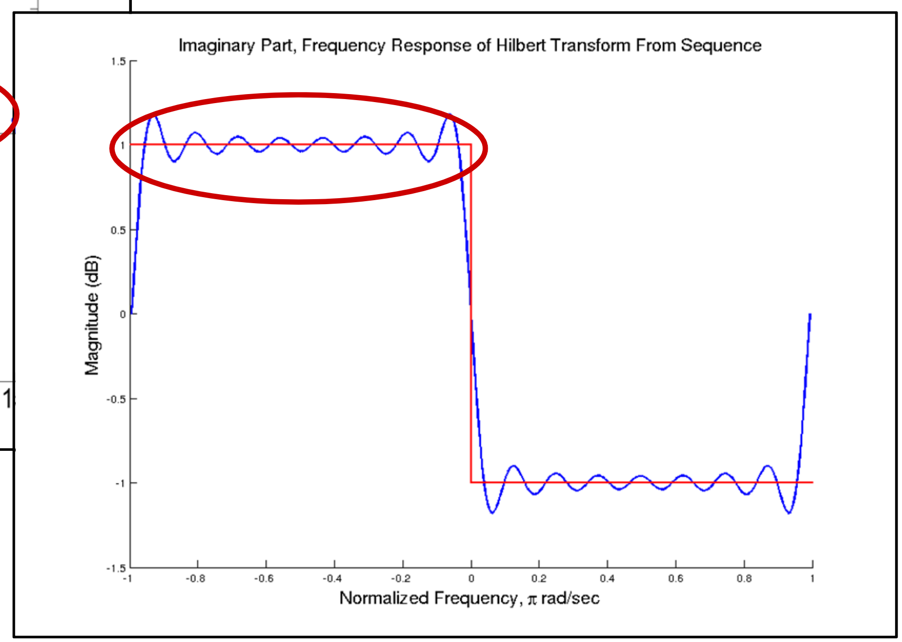
Building Filter from Discrete Sequence

- h[n] = 2/(πn) for n even
- h[n] = 0 for n odd



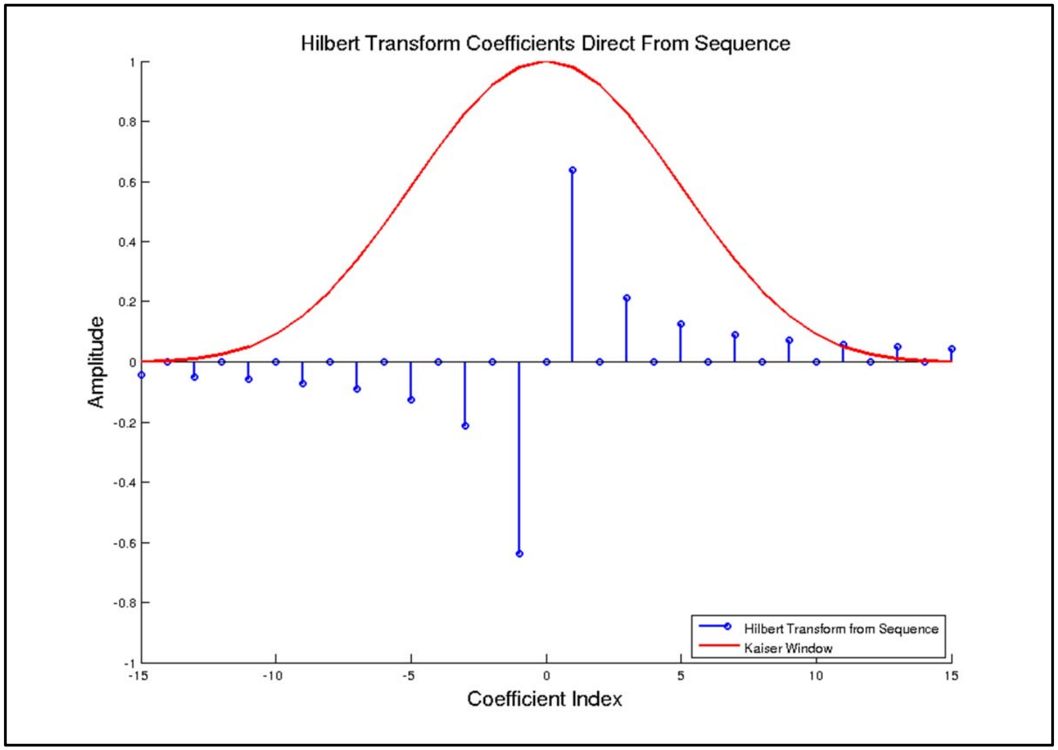
Reducing Ripple

- Ripple due to Gibbs' Phenomenon
- Window coefficients to combat ripple

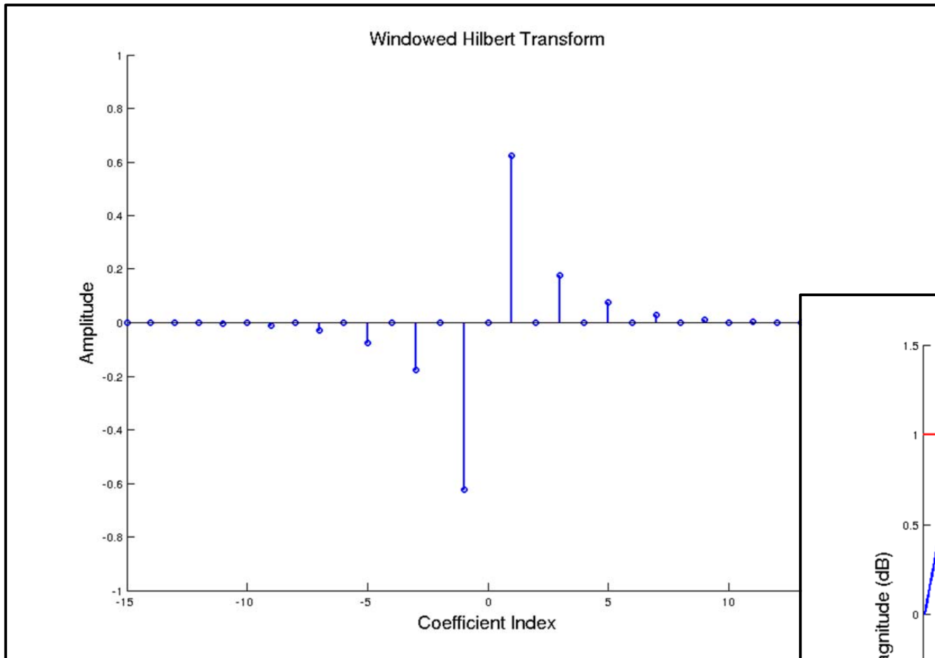


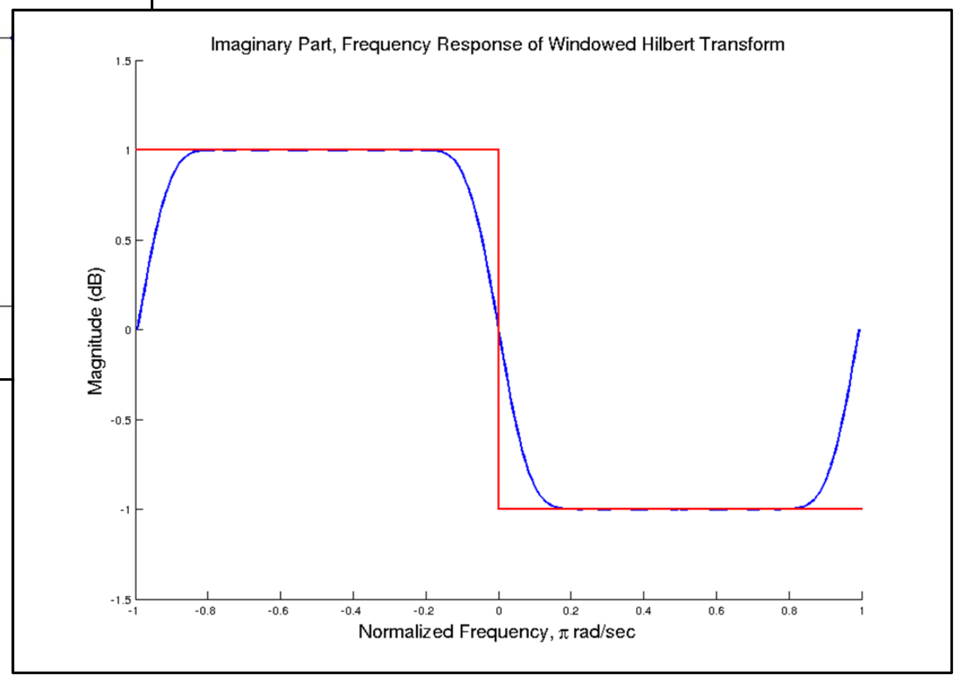
Reducing Ripple (Con't)

- Force tails of filter to zero artificially through windowing



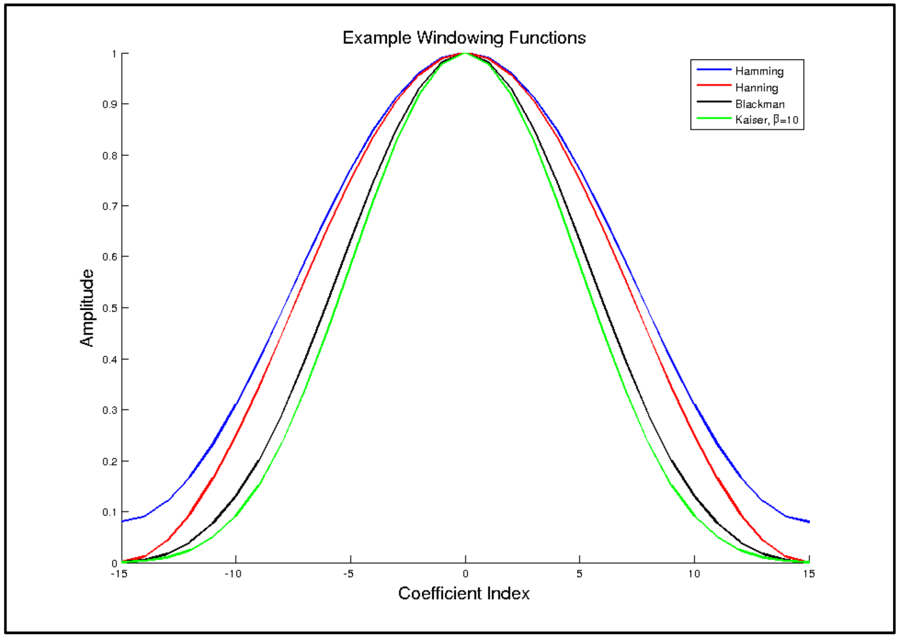
Reducing Ripple (Con't)





Change Design Method

- Choosing ‘best’ window is difficult
- Instead of designing in time, design in frequency



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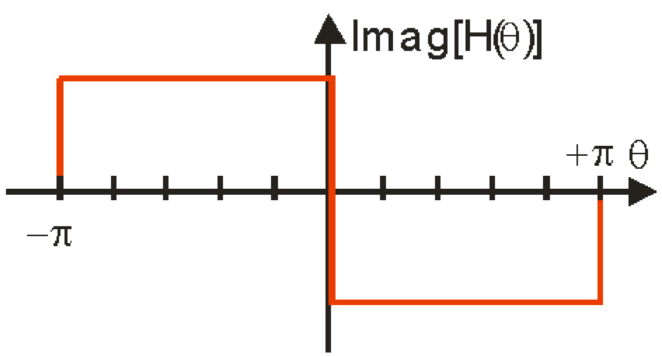
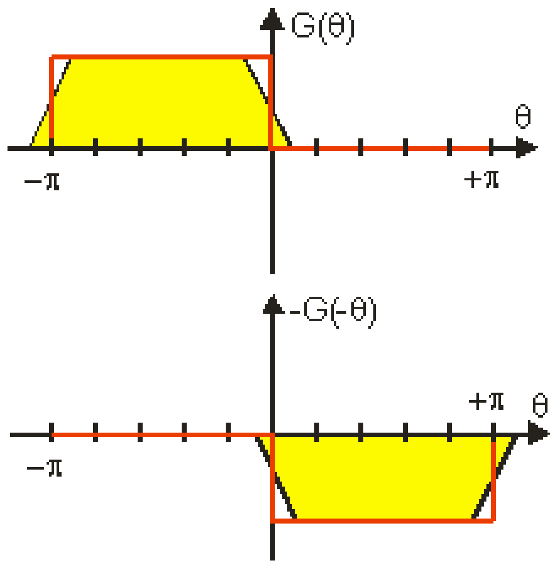
Hilbert Transform Frequency Response

- By definition:
  - H(w) = -j sgn (w)
  - Approximate with two half band filters
- How to build a half band filter?





Half Band Filter Design

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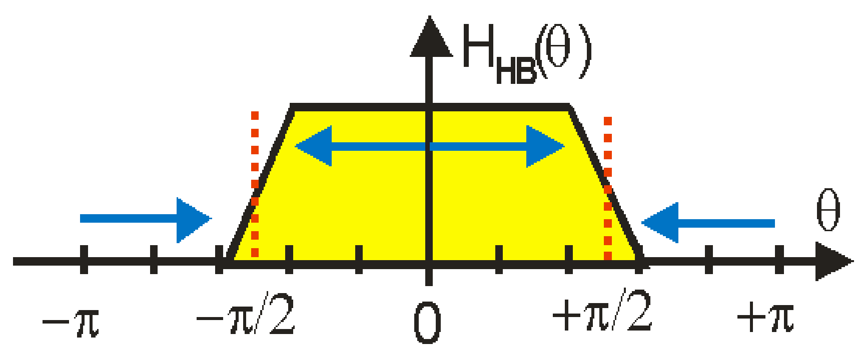
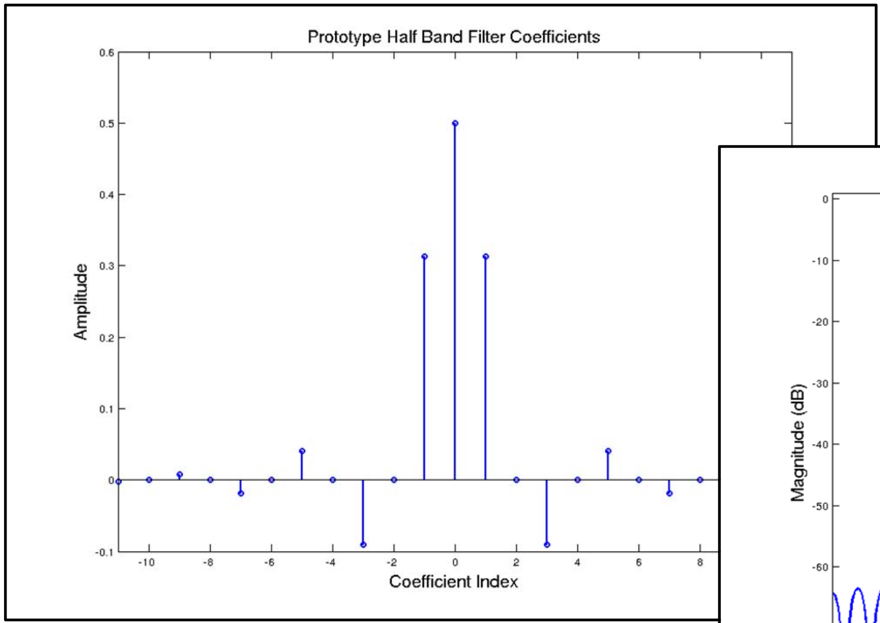
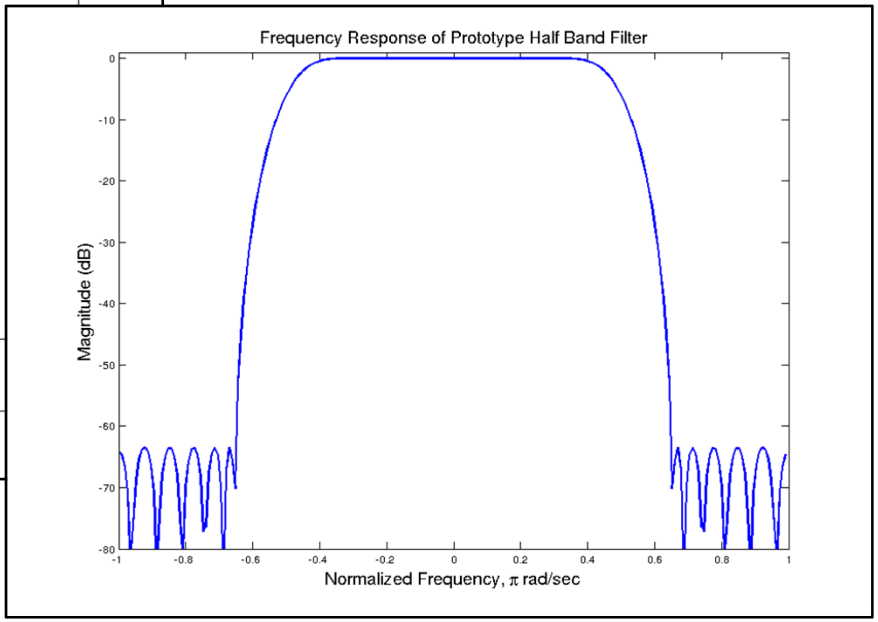
- A half band filter, filters half the spectrum
- Every other coefficient is zero
- Quick design method (MATLAB code);
  - f = [0 wc 1-wc 1];
  - a = [1 1 0 0];
  - hb = firpm(N-1,f,a);



Half Band Filter Design (Con't)

- Coefficients have ‘zeros’ every other sample
- Frequency response covers appropriate band

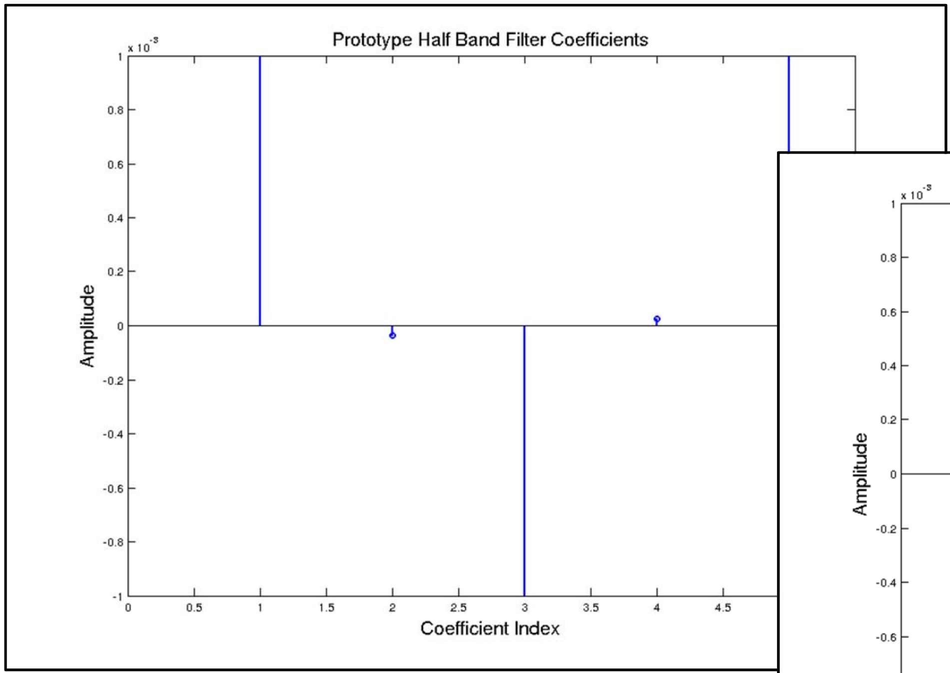


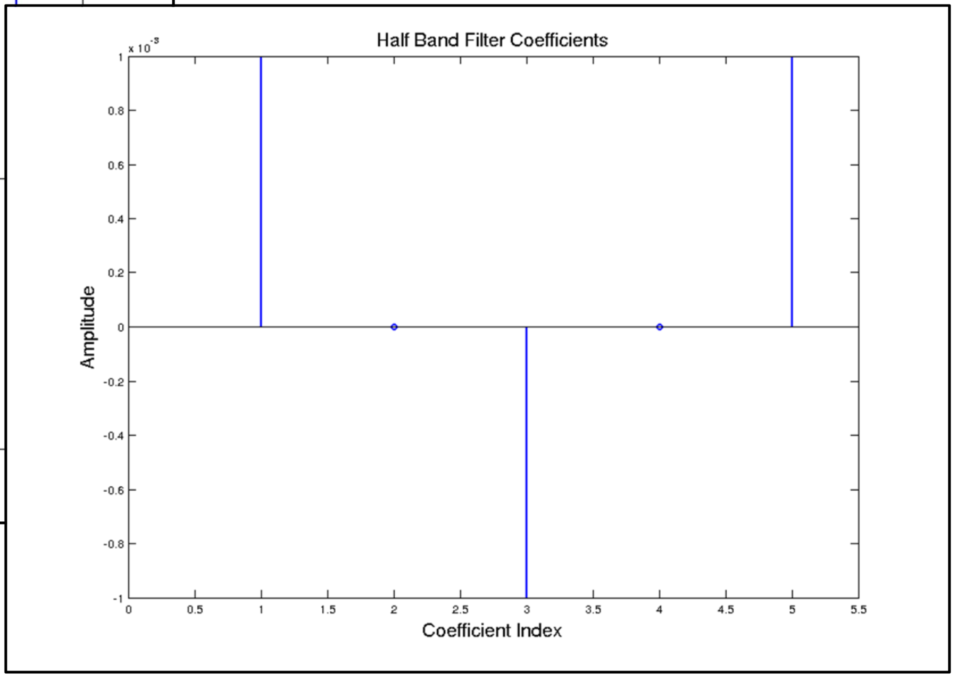


Half Band Filter Design (Con't)

- Parks-McClellan doesn't set zero coefficients to exactly zero

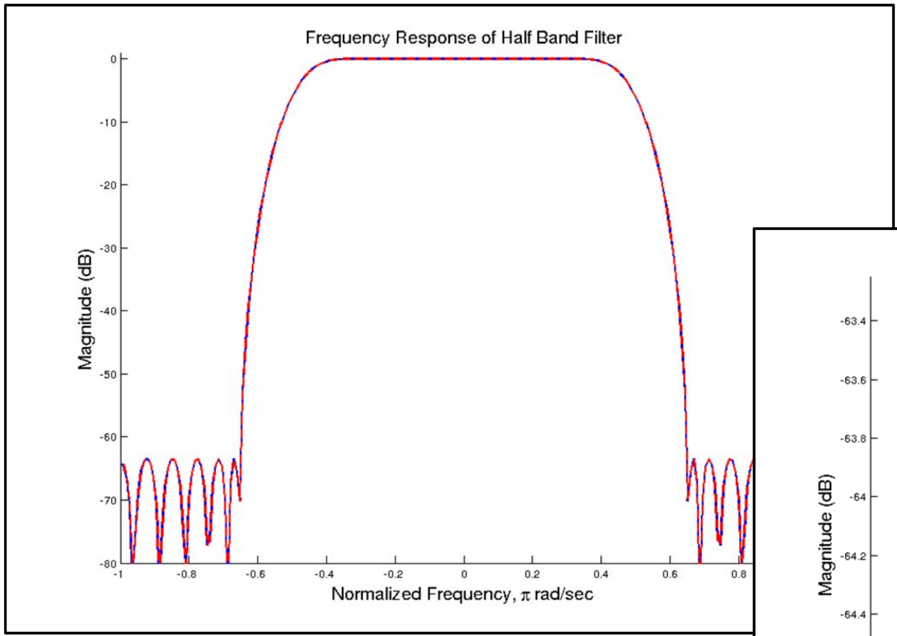
- Force coefficients to zero

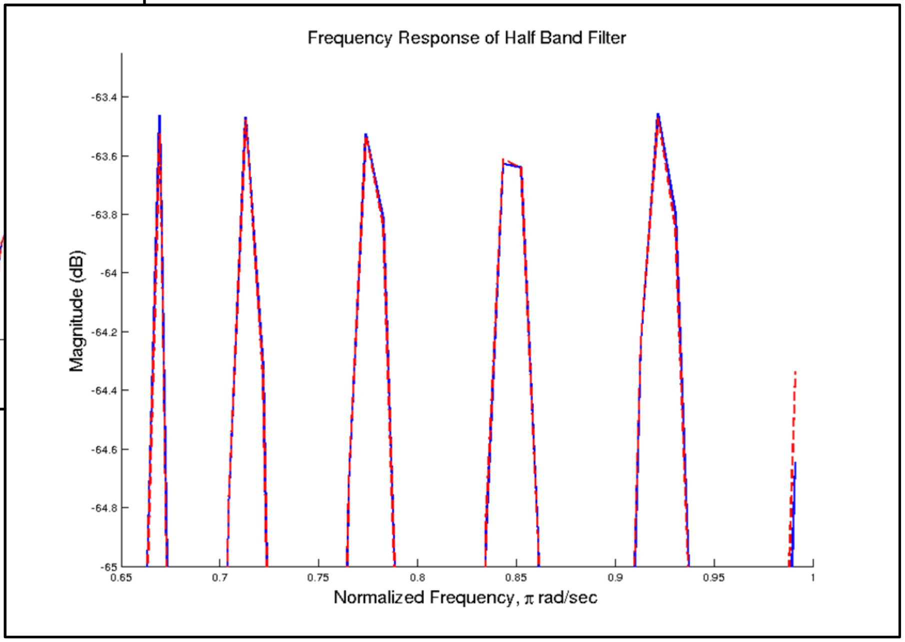




Half Band Filter Design (Con't)

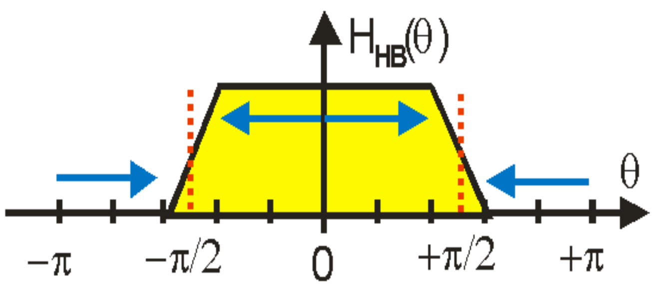
- Change in frequency response is negligible

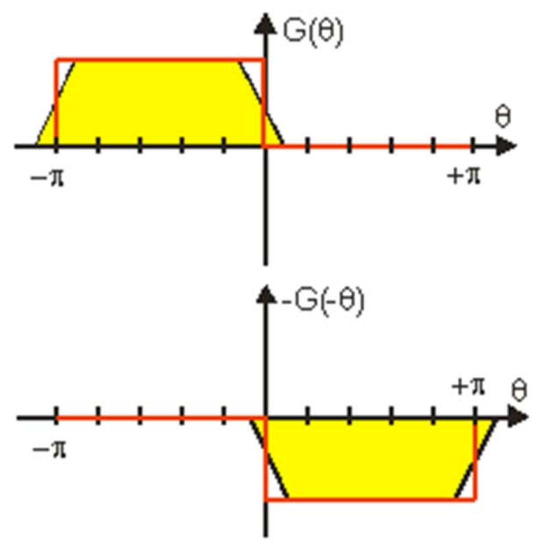




Sum Half Band Filters

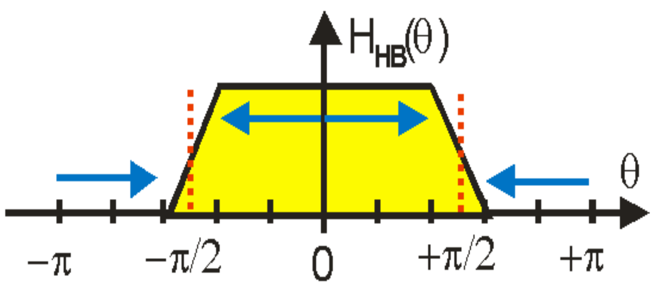
- $G(\theta) = H_B(\theta - \pi/2)$
- $-G(-\theta) = - H_B(\theta + \pi/2)$
- $H_{HT}(\theta) = j\ (\ G(\theta) + G(-\theta)\ )$
- $H_{HT}(\theta) = j\ (\ H_{HB}(\theta - \pi/2) - H_{HB}(\theta + \pi/2)\ )$

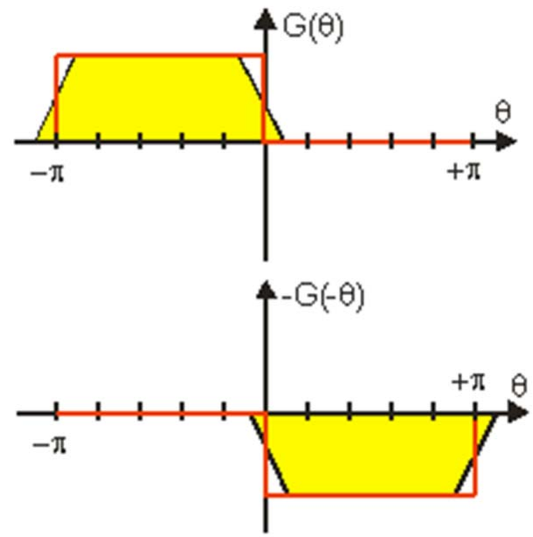




Sum Half Band Filters (Con't)

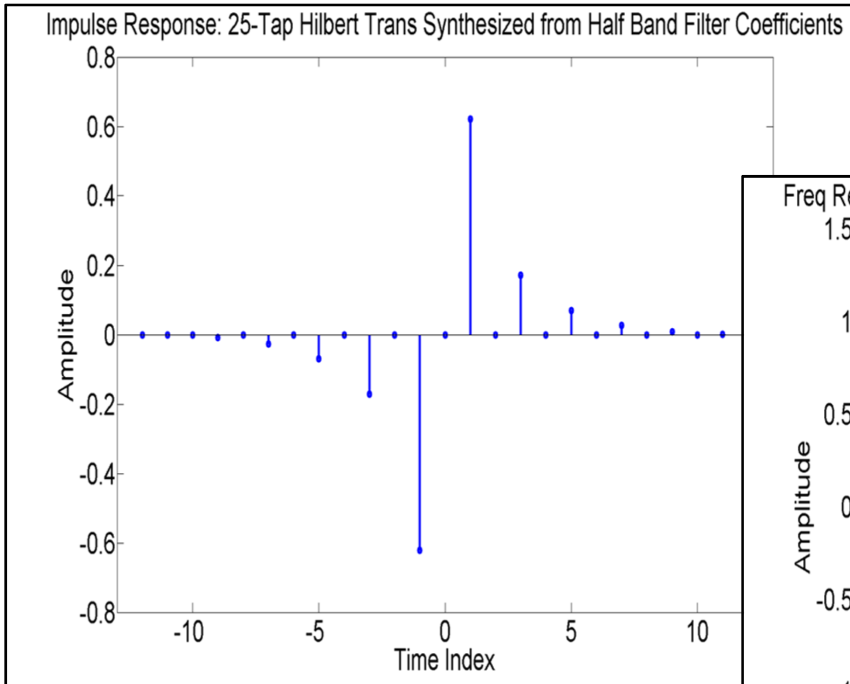
- $H_{HT}(\theta) = j\,(\,H_{HB}(\theta - \pi/2) - H_{HB}(\theta + \pi/2)\,)$
- $H_{HB}(\theta - \pi/2) \leftrightarrow j\, h_{HB}[n]\exp(-j\pi n/2)$
- $H_{HB}(w + \pi/2) \leftrightarrow j\, h_{HB}[n]\exp(j\pi n/2)$
- $h_{HT}[n] = 2\, h_{HB}[n]\,\sin(\pi n/2)$

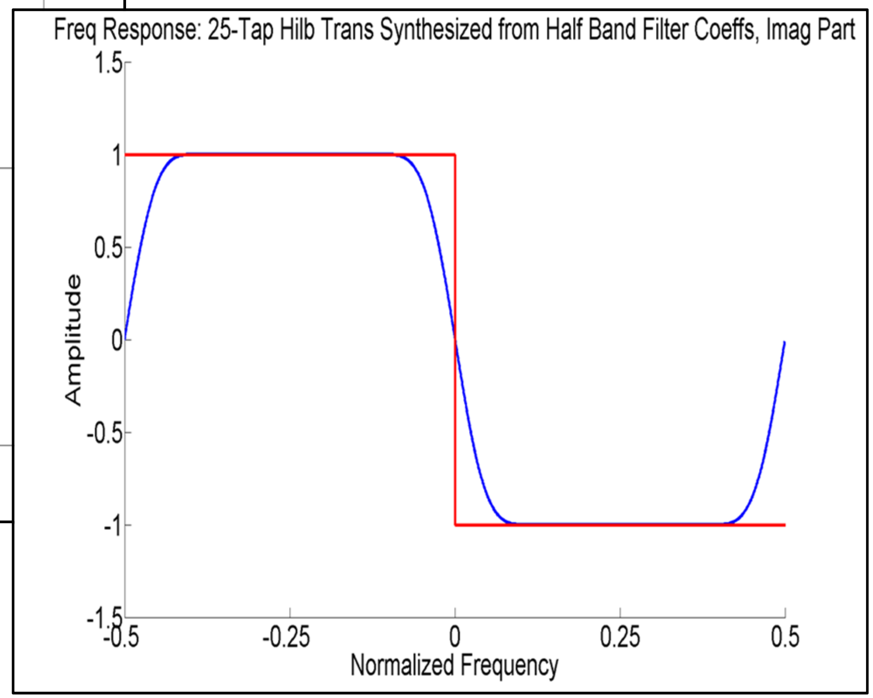




Hilbert Transform Coefs from Half Band Coefs

- $h_{HT}[n] = 2\ h_{HB}[n]\ \sin(\pi n/2)$





Hilbert Transform Coefs from Half Band (Con't)

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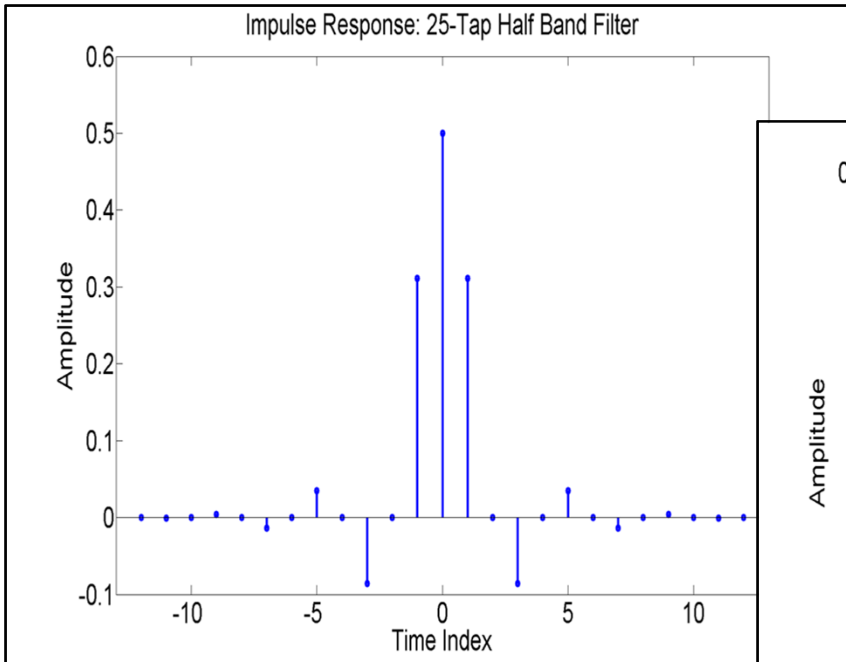
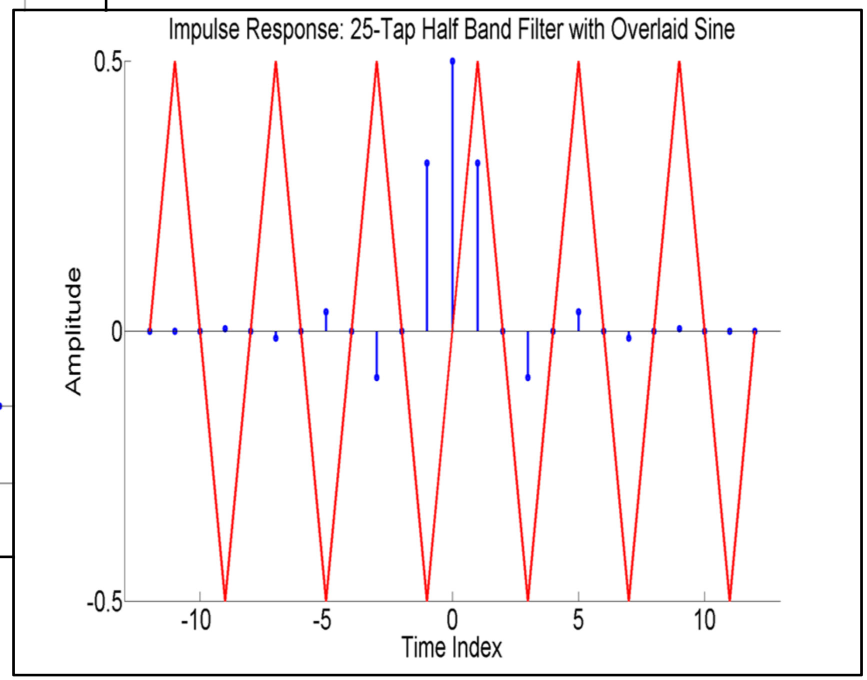
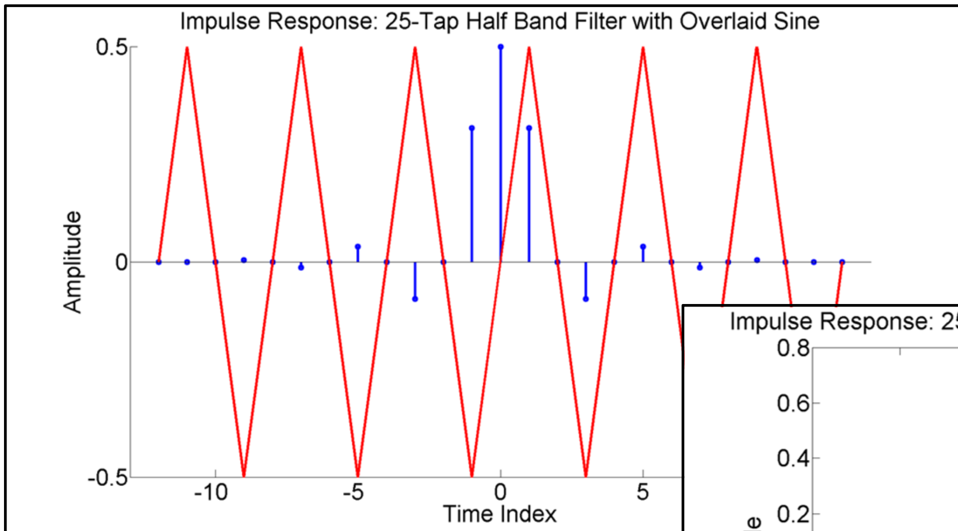
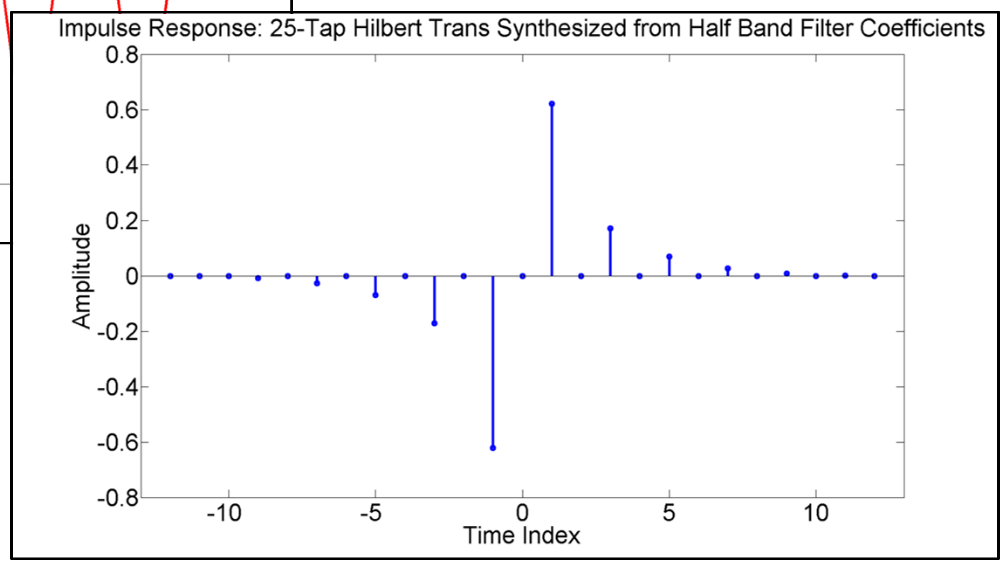




Hilbert Transform Coefs from Half Band (Con't)

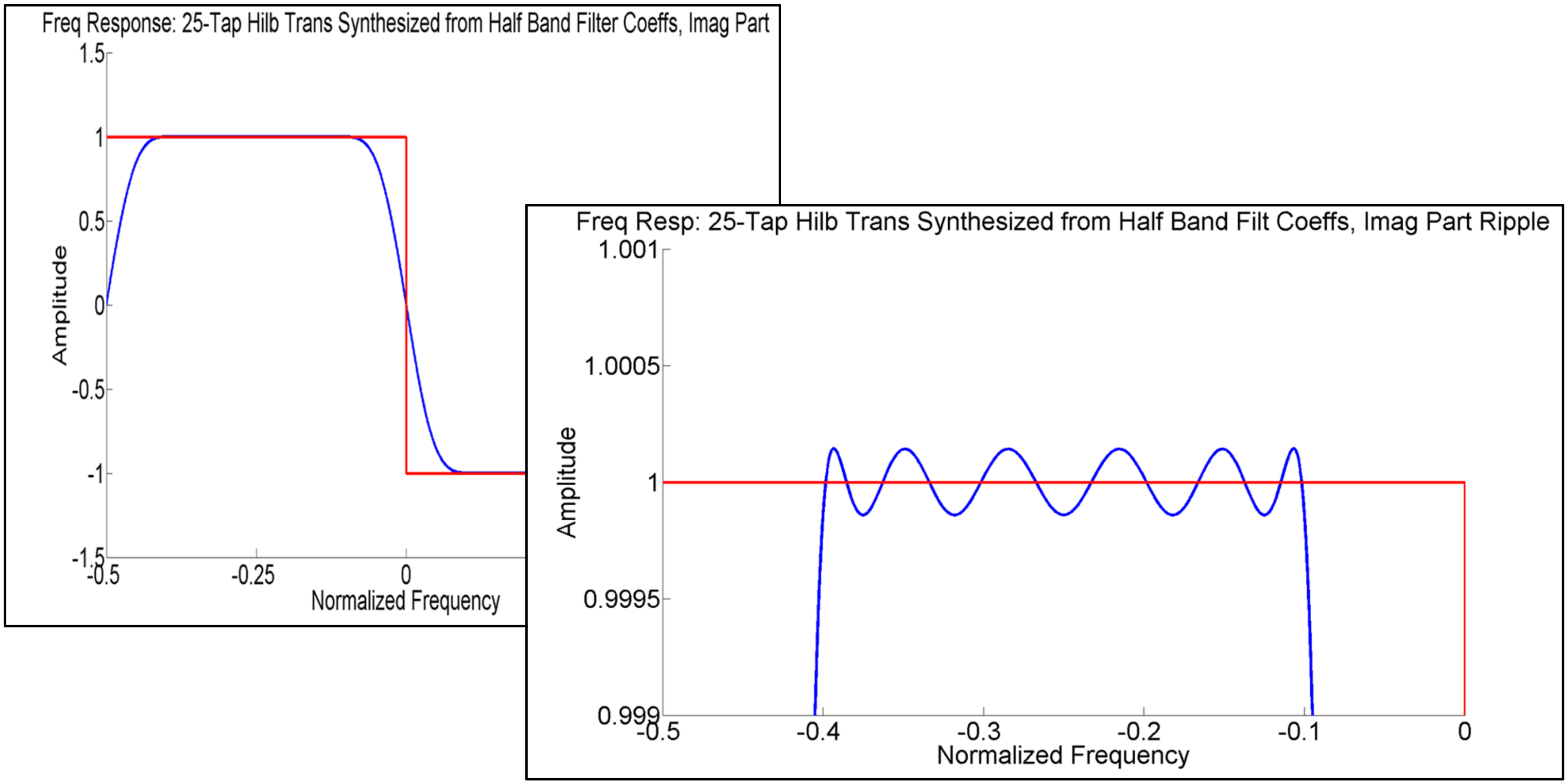
- Sine wave applies Hilbert transform filter properties





Hilbert Transform Filter Response

- Greatly improved passband ripple



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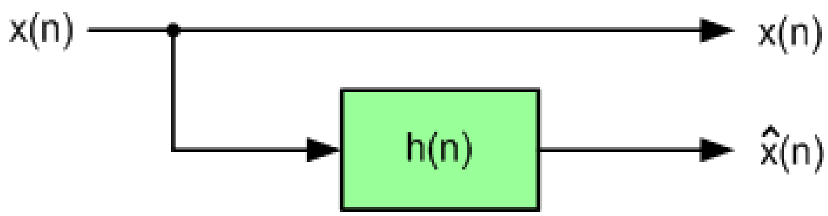
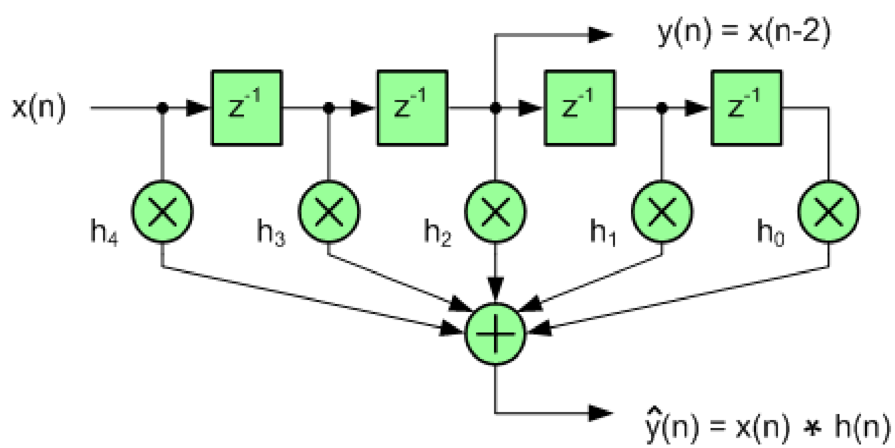
Implementation

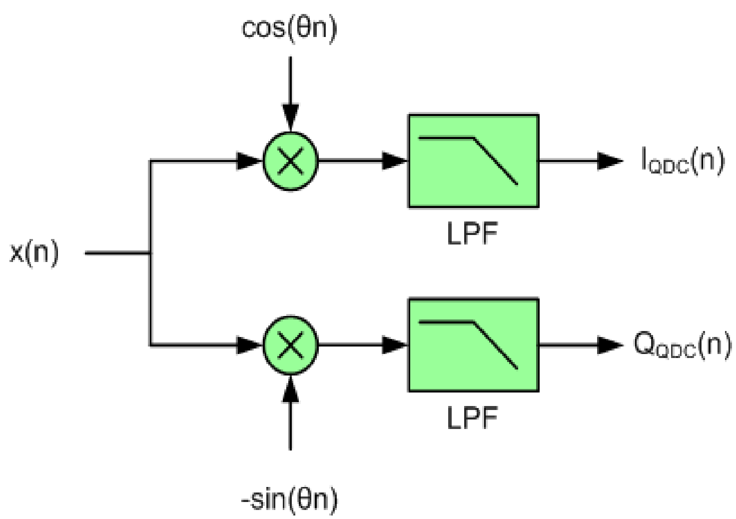
- Hilbert Transformer operates on imaginary portion
- Delay real portion accordingly

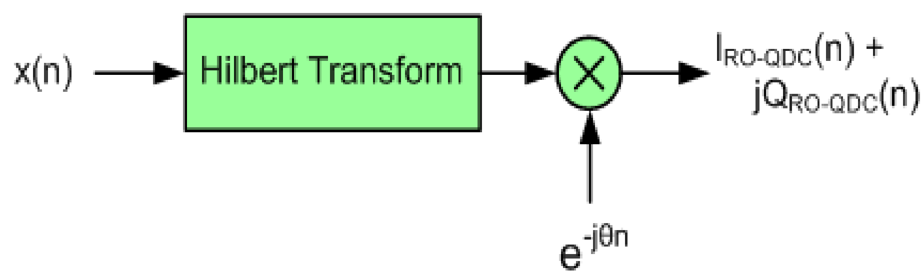




Total Computations Required

- Quadrature Downconverter
  - Two low pass filters of order N, two multiplies
- Downconversion with Hilbert Transformer
  - One filter of order N, one complex multiply





Conclusion

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- Compared quadrature downconverter and downconversion with Hilbert transformer
- Reviewed Hilbert transform
- Discussed windowing Hilbert transform filter
- Designed Hilbert transform filter in frequency
- Covered Design process for half band filter
- Results
- Implementation

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