POLYPHASE FILTER BANK FOR UNEQUAL CHANNEL BANDWIDTHS AND ARBITARARY CENTER FREQUENCIES

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ABSTRACT

Traditional M-Path channelizers are multichannel filter banks with identical bandwidths and equal spaced center frequencies. Many applications desire channelizers with unequal channel bandwidths and with non-equally spaced channel spacing. A simple example is a cable plant which may have a mix of 6 MHz and 8 MHz channels distributed over its frequency span with select odd width channel slots interspersed in the frequency band at specific legacy center frequencies. We have developed two methods that permit a modified channelizer to accommodate baseband shaped channels of any bandwidth and position them at arbitrary center frequencies. We report on the two methods here.

1. INTRODUCTION

A basic synthesis channelizer or multi-channel up-converter is formed by an M-Point IFFT, an M-Path polyphase filter, and an M-port output commutator. This common structure is shown in Figure 1.

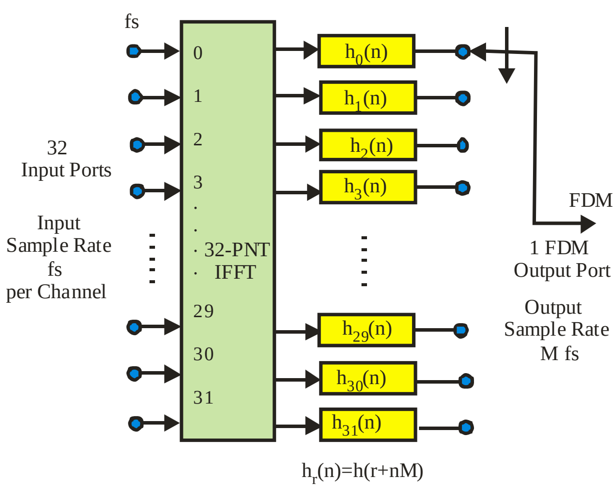

Figure 1. M-Path Polyphase Synthesis Channelizer

Parallel shaped and sampled data streams are delivered via an M-point input vector to the IFFT at the common input sampled rate of f_S. Each sample of the input time series presented to the k-th bin of the IFFT contributes a scaled complex sinusoid of k-cycles per M-sample length output vector. These samples are shaped by the polyphase filter weights, summed with previous weighted output vectors and then sequentially presented to the single output port at an output sample rate M-times the input sample rate or $M{\cdot}f_S$. This channelizer has equally spaced center frequencies at multiples of the input sample rate. We consider a specific example, as shown in Figure 1, of a 32-path channelizer designed for 8 MHz center frequencies with input sample rates of 8 MHz which forms an output sequence with sample rate of 8*32 or 256 MHz. The individual channel spectra are translated replicas of the prototype low-pass filter spectrum prior to its partition into the M-path filter weights. Figure 2 shows three possible relationships between channel bandwidth and channel spacing. The options are seen to be channel bandwidth less than, equal to, or greater than the channel spacing. Our interest in this paper is the last two options. To satisfy the Nyquist criterion for these wider bandwidth channels we require that the input sample rate be greater than channel spacing. For the process we describe here we modify the channelizer to accept input samples at $f_S=4f_C$ while preserving the channel spacing at f_C as opposed to f_S. We motivate this modification and show the modification that supports it in the next section.

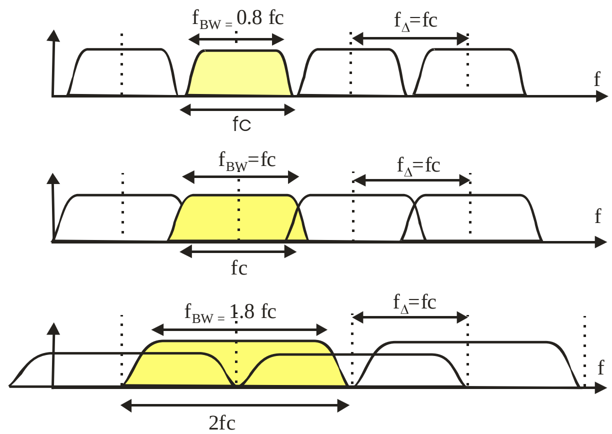

Figure 2. Some Possible Channel Widths Relative to Channel Spacing

2. MOTIVATION TO MODIFY CHANNELIZER

The basic channelizer up-converts input signal spectra from baseband to one of the multiples of the channel spacing frequencies $k \cdot f_C$. The central idea of the modified channelizer is a spectral translation of the baseband signals presented to the channelizer. Suppose we examine a baseband signal and its spectral image at frequency $k_C \cdot f_C$. If we use a complex heterodyne to displace the baseband spectrum by $\pm\Delta f$ then the spectral position of its spectral image is also shifted by $\pm\Delta f$ to $k_C \cdot f_C \pm \Delta f$. This translation is visualized in Figure 3. What we accomplish is a dual conversion translation, one by the low sample rate heterodyne and one by the polyphase channelizer. We have likened this to a worm hole: we turn the baseband heterodyne knob and its spectral image shifts along with the baseband shift.

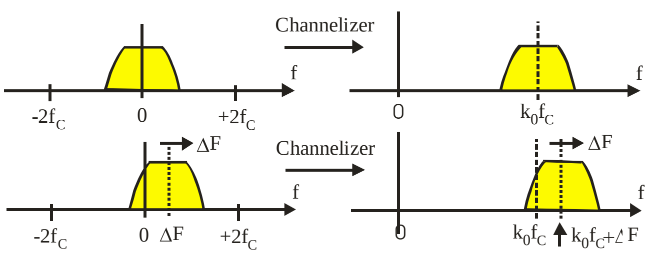

Figure 3. Baseband Signal and Its Channelized Image and Offset Baseband Signal and its Offset Channelized Image

The allowable range of baseband frequency offset is shown in Figure 4. Here we see that the maximum bandwidth signal is confined to the interval bounded by $\pm f_C$ and that to accommodate reasonable transition band edges of the shaping filter in the channelizer the input sample rate to the channelizer is selected to be twice the 2-sided BW or $4f_C$.

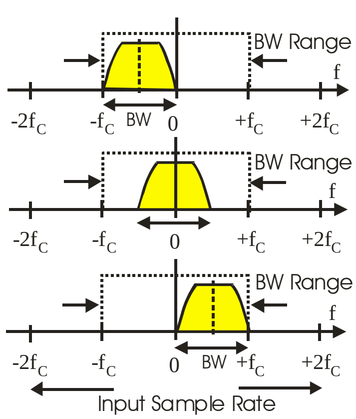

Figure 4. Baseband Frequency Offset Range

The frequency range and bandwidths of the channelizer translated image is shown in Figure 5. We first note, in the configuration suggested in this figure, that the channel bandwidth is twice the channel spacing and overlaps by 50%. Note that at the maximum positive frequency offset from the nominal center frequency of the top segment of this figure the bandwidth in this channel filter exactly matches the bandwidth with maximum negative frequency offset in the next highest channel filter. A similar relationship exists for the maximum negative frequency offset in this channel filter and the next lower channel filter.

Thus we can shift the input bandwidth to the positive edge of a particular center frequency band and can slide past that edge by transferring the signal to the negative edge of the next adjacent center frequency band.

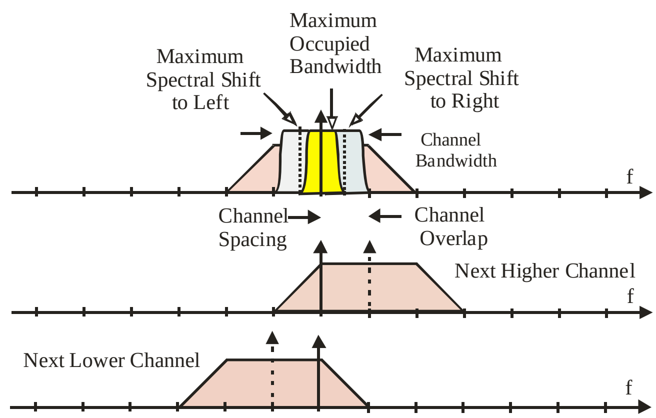

Figure 5. Carrier Centered Frequency Offset Range and Overlapped Frequency Intervals with Adjacent Channel Filters

3. MODIFIED CHANNELIZER

We now modify the channelizer to accept input signals at sample rate $4f_C$ while preserving the channel spacing of f_C. Following our example of 8 MHz channel spacing, the required input sample rate would be 32 MHz. The shaping filters augmented with arbitrary interpolating filters supply the various bandwidth signals to the channelizer at this fixed output sample rate. Since the input sample rate to the channelizer is already 4-times the nominal channel spacing, the channelizer is reconfigured to up-sample by M/4, i.e. 32/4 rather than by M or 32. We accomplish this by up-sampling by 32 in the IFFT and then down-sampling by 4 in the polyphase filter. The channelizer center frequencies are maintained at multiples of f_C or 8 MHz, frequencies determined by the M-point IFFT and a circular buffer between the IFFT and the M-path filter. The structure of the modified channelizer is shown in Figure 6.

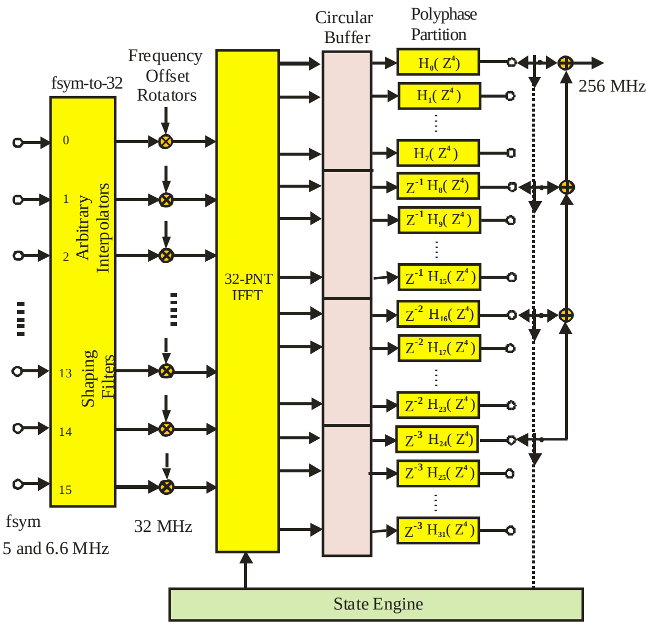

Figure 6. Modified Channelizer Complex Rotators, M-Point IFFT, Circular Buffer, M-Path Polyphase Filter, and 4-to-1Path Combiner

Equation 1 shows the Z-transform of the band-pass or heterodyned version of the prototype low-pass filter embedded in the M-path channelizer. Equation 2 presents the Z-transform of the dual sum or polyphase partitioned version of the same band-pass filter. Note the partition contains polynomials in Z^M which enables us to apply the noble identity to the M-path filter and thus operate the M-path filters at the lower of the input and output sample rates. The delays in the final sum guide the output commutator, and the phase rotators aligned with the delays are performed by the IFFT.

$$H(Z) = \sum_{n=0}^{N-1} h(n) e^{j\frac{2\pi}{M}kn} Z^{-n} \tag{1}$$

$$\begin{aligned} H(Z) &= \sum_{r=0}^{M-1}\sum_{n=0}^{N-1} h(r+nM) e^{j\frac{2\pi}{M}k(r+nM)} Z^{-(r+nM)} \\ &= \sum_{r=0}^{M-1} Z^{-r} e^{j\frac{2\pi}{M}kr} \sum_{n=0}^{N-1} h(r+nM) Z^{-nM} e^{j\frac{2\pi}{M}knM} \\ &= \sum_{r=0}^{M-1} Z^{-r} e^{j\frac{2\pi}{M}kr} H_r(Z^M) \end{aligned} \tag{2}$$

Equation 3 shows the modification of equation 2 required to support the 1-to-M/4 re-sampling rather than the 1-to-M re-sampling. Here we see the polyphase polynomials in $Z^{M/4}$, the frequency dependent phase shift due to the M/4 re-sampling, and the additional 4-point delayed sum which performs the final 4-to-1 down-sampling. These extra delays are absorbed in the polyphase filter partition. Figure 7 shows the structure of the modified polyphase filter partition. The frequency dependent phase shift is inserted under control of the state engine by the circular buffer which takes advantage of the equivalency of time delay and frequency dependent phase shift.

$$\begin{aligned} H(Z) &= \sum_{r=0}^{M-1}\sum_{n=0}^{N-1} h(r+nM) Z^{-(r+nM)} e^{j\frac{2\pi}{M}k(r+nM)} \\ &= \sum_{r=0}^{M-1} Z^{-r} e^{j\frac{2\pi}{M}rk} \sum_{n=0}^{N-1} h(r+nM) Z^{-nM} \\ &= \sum_{s=0}^{3}\sum_{r=0}^{\frac{M}{4}-1} Z^{-(r+s\frac{M}{4})} e^{j\frac{2\pi}{M}(r+s\frac{M}{4})k} \sum_{n=0}^{N-1} h(r+nM) Z^{-n\frac{M}{4}} \\ &= \sum_{s=0}^{3} Z^{-s\frac{M}{4}} e^{j\frac{2\pi}{4}sk} \sum_{r=0}^{\frac{M}{4}-1} Z^{-r} e^{j\frac{2\pi}{M}rk} \sum_{n=0}^{N-1} h(r+nM) Z^{-n\frac{M}{4}} \end{aligned} \tag{3}$$

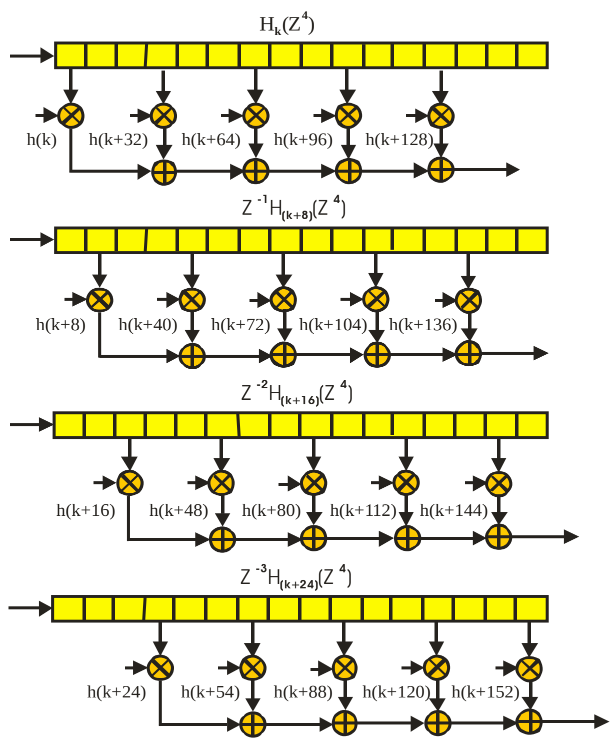

Figure 7. Details of Modified Polyphase Filter Partition

We have chosen not to meticulously develop the sequence of operations that form the architecture of the modified polyphase filter partition shown in Figure 7. We have developed that sequence for a 1-to-M/2 channelizer in a recent paper [6] and suggest the interested reader follow the details presented there.

4. PROTOTYPE LOW-PASS FILTERS

We commented in the introduction that the prototype filter can be implemented in two different realizations that correspond to the second and third filter options presented in Figure 2. These options permit two different bandwidth filters: one equal to the channel spacing, and one equal to twice the channel spacing. We off handedly introduced the second option in Figure 5. There we recognized that the wider, hence overlapped, bandwidth of the double bandwidth prototype filter permitted frequency offsets we introduced at baseband to be contained within the channelized bandwidth at each center frequency formed by the channelization process. The relationship between the offset spectra and the frequency response of double bandwidth prototype filter is illustrated in Figure 8.

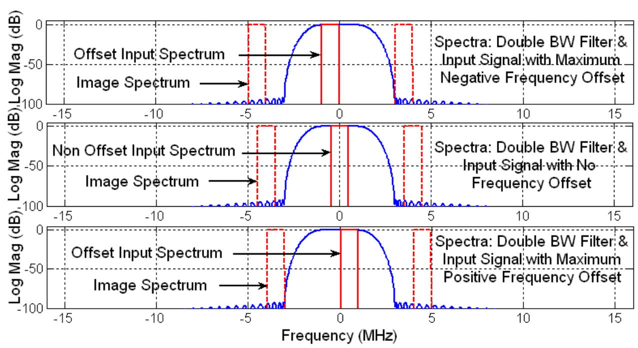

Figure 8. Spectra of Double Bandwidth Prototype Low-Pass Filter and Spectra of Offset Input Spectrum

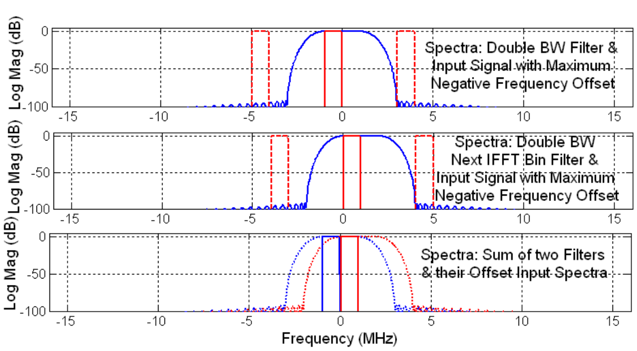

Figure 9. Spectra of Double Bandwidth Adjacent Frequency Bin Filters and their Offset Input Spectra

We also recognized that adjacent filters with bandwidth twice the channel spacing overlap by 50% and that the overlapped filter coverage by adjacent filters in the filter bank permitted soft handoff between adjacent channel bands as we slide the input spectrum through the frequency bands covered by the channelizer. Figure 9 shows the spectral response of adjacent channels and the spectra of their frequency offset input signals as well as their sum formed by the channelizer. Note that the offset spectra reside on either side of zero frequency (DC). This process can form channelizers with an even number of channels without a channel centered on DC. The standard channelizer cannot do this.

The second filter option is a single bandwidth Nyquist filter designed for 6 dB crossover gains between adjacent channels. This filter is designed with the Remez algorithm with the stop band edge shifted away from the channel cross over frequency till the two adjacent filters cross the midpoint frequency at precisely $20\log_{10}(0.5)$ or -6.0206 dB. Figure 10 shows the frequency response of a Nyquist filter designed for the 32 channelizer along with a zoom to the crossover frequency between the adjacent frequency filter.

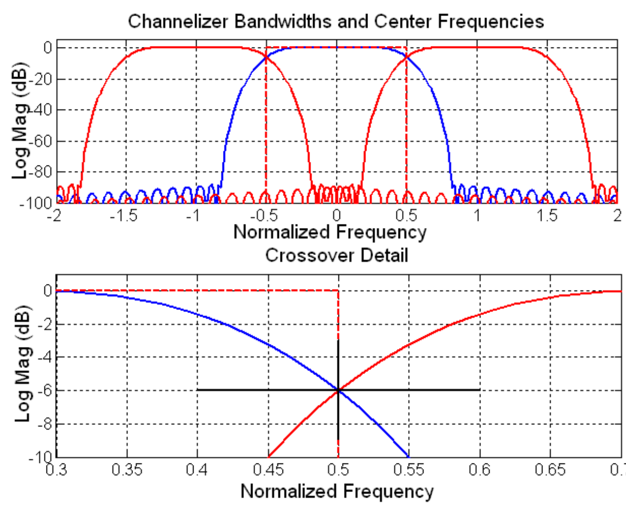

Figure 10. Frequency Response of Adjacent Frequency Nyquist Filters with Detail of Crossover Region

The channelizer operating with this filter in its conventional mode can place signal spectra at selected center frequencies of the channelizer. The clever part starts now. Because we modified the channelizer to operate at an input sample rate of $4 \cdot f_C$ the sample rate permits spectral translation of the input spectrum by as much as $\pm f_C/2$ by a baseband heterodyne. The amount by which we shift the baseband spectrum Δf (say +2 MHz) is the amount by which the spectrum is also shifted in the specific up-converted channel of the channelizer. Of course this shift results in part of the spectrum being rejected by the channel bandwidth of the channelizer. We know this! We respond by placing the same spectral offset to the same frequency band in the adjacent channel centered f_C above our specific channel. For our example we offset the same signal by -6 MHz in the adjacent channel located 8 MHz above us. This channel too will have part of its spectrum rejected by the bandwidth of the channel filter. Since the channel filters are Nyquist filters crossing at their 6 dB levels the sum of the spectra carried by the adjacent channels is the assembled version of the input spectrum. Figure 11 shows the dual heterodyne of the offset input signal into the channel bands centered at frequency index k and k+1.

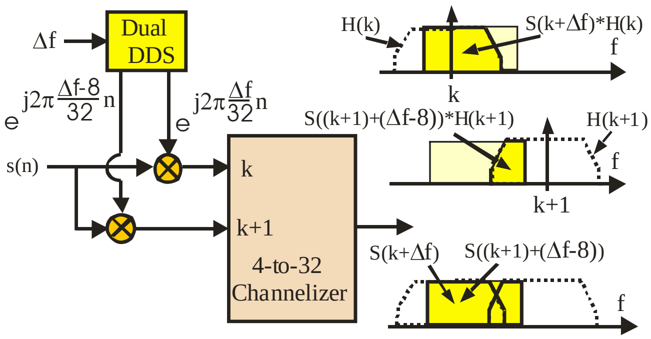

Figure 11. Dual Heterodyne of Offset Signal Spectrum into Adjacent Channels of 32-Path Channelizer

Figure 12 shows the input spectra of a single input signal placed at the same output frequency in two adjacent baseband channels with a positive offset Δf in ch(k) and a negative offset -8+ Δf in ch(k+1) with 8 MHz offset relative to ch(k). We then see the individual spectral rejection due to the two channel filters when up-converted one channel at time as well as the sum of the spectra due to the sum of the two channels filters in the channelizer when up converted simultaneously. The spectral responses of the adjacent channels are the offset Nyquist filters crossing at their -6 dB gain levels. The sum of the adjacent channels perfectly reconstructs the original spectrum from the spectral fragments.

Figure 13 shows the spectrum of a multichannel channelizer with various equal bandwidth channels positioned at arbitrary frequency offsets from their nominal center frequencies. This figure is the output of a versatile multi channel signal generator and digital up-converter (DUC) channelizer MATLAB simulation.

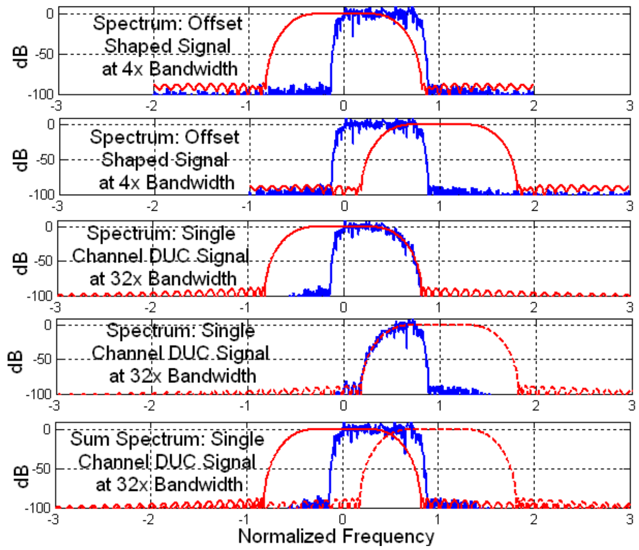

Figure 12. Spectra of Offset Baseband Signals into Adjacent Channels in Channelizer Showing Separate Channel Responses and the Combined Channels Response

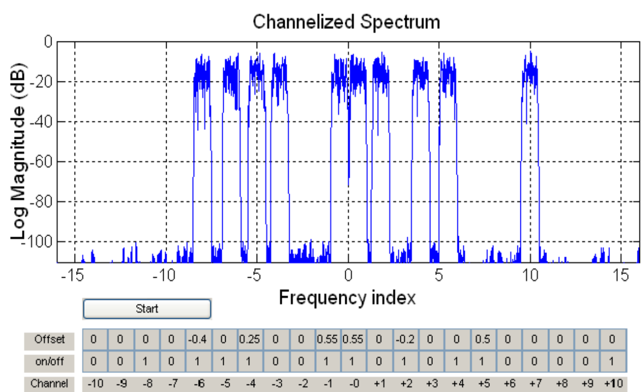

Figure 13. Spectrum of a Multichannel Channelizer with Various Channels Enabled and Offset from their Nominal Positions by Arbitrary Offsets from Nominal Center Frequencies

5. CLOSING COMMENTS

We have described a variant of the standard polyphase synthesis channelizer that supports frequency translation of the various input signal spectrum by baseband heterodynes prior to presentation to the channelizer. To accommodate the additional bandwidth occupied by the frequency translated input signals, the channelizer expects the input signals to be delivered at higher sample rates than required to satisfy the Nyquist criteria for the non-shifted spectrum. Since the equivalent two sided bandwidth is doubled by the maximum permitted frequency shift we must expect at least a doubling of the input sample rate. Since many systems prefer to operate at 2-samples per symbol or 2-samples per bandwidth, the doubling of the sample rate leads us to operate the input and the channelizer at 4-samples per bandwidth. Operating the channelizer at 4-samples per bandwidth offers us a second option; that of raising the input signals bandwidth by the same factor of 2. We presented in this paper the modified polyphase partition and the added circular buffer to accommodate the change in the filter structure that converted the basic 1-to-M up sampling channelizer to a 4-to-M up sampling channelizer.

We demonstrated that the new architecture of the channelizer could accommodate two different low-pass filter prototypes. The first prototype operates with a 0.1 dB bandwidth equal to the twice the channel spacing and the second prototype operates with a 6-dB bandwidth equal to the channel spacing. The two filters perform the same function by different approaches. For comparison, the double bandwidth filter has a significantly wider transition bandwidth than does the single bandwidth filter which leads to prototype filter lengths of 64 samples and 256 samples respectively. These lengths, in turn, form the 32 polyphase path filters with 2-taps or 8-taps respectively. Both filters are quite short considering the task they perform: channelizing multiple channels in a 32-path filter bank.

A final comment on the variant of polyphase filter presented in this paper is called for. The channelizer we presented is a DUC, or synthesizer channelizer. The dual of this system would be the digital down-converter (DDC), or analysis channelizer. For all practical considerations, the two channelizers require the same resources and perform the same tasks but in opposite orders. We have designed a number of analysis channelizers to accommodate multi-channel, mixed bandwidth, arbitrary center frequency signal sets. They are always fun to implement and are always impressive in terms of small computational burden per output channel time series.

6. REFERENCES

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