

# SYNTHETIC SYMMETRY IN COGNITIVE RADIO NETWORKS

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## ABSTRACT

In [1] we introduced distributed non-cooperative dynamic frequency selection (DFS) algorithms which use only locally available information but achieve performance equivalent to centrally planned omniscient algorithms. These algorithms require the network to satisfy a condition termed *Bilateral Symmetric Interference* (BSI). While BSI holds between pairs of equal-power devices for most waveforms, establishing the existence of BSI across networks is difficult.

In this paper, we show how to establish BSI in infrastructure and ad-hoc networks, in networks employing power control, and in networks which prioritize the performance of different users. When these techniques are combined with similar techniques developed by CRT for multi-carrier multi-antenna systems, most networks running most waveforms can achieve optimal allocations of layer 1 and 2 resources with low-complexity distributed algorithms using only locally available information.

## 1. INTRODUCTION

In general, algorithms which appear attractive for a single cognitive radio (CR) can lose their luster when deployed in a network. For instance, interference avoidance can dramatically improve the performance of a link and lies at the heart of many proposed CR applications. But now consider the system of three coexisting, but uncoordinated, wireless links {1,2,3} depicted in Figure 1 which connect three access nodes (AN) to three clients. In this interference avoidance scheme each AN chooses between two orthogonal channels, {0,1}, to minimize the interference its client experiences from the other ANs (presumably the clients are sufficiently separated to be effectively noninterfering). Let us then suppose that  $g_{31} > g_{21}$ ,  $g_{12} > g_{32}$ ,  $g_{23} > g_{13}$  where  $g_{ij}$  is the link budget gain (path loss) from the AN of link  $i$  to the client of link  $j$ . In other words, client 1 is interfered with more by AN 3 than by AN 1; client 2 is interfered with more by AN 1 than by AN 2; and client 3 is interfered with more by AN 2 than by AN 1. Without a loss of generality, assume  $g_{31} = g_{12} = g_{23} = 1.0$  and  $g_{21} = g_{32} = g_{13} = 0.5$  and a transmit power of 2 so the observed interference at each client is equal to twice the sum of path gains from the other links' ANs to the client.

In this two channel system, there are eight ( $2^3$ ) different channel allocations which could be made by the independent choices of the three AN. For these eight combinations, the interference levels experienced by each client is shown in

Table 1 where the entries on Channel-labeled rows specify the choice of channels by each AN (1,2,3) and Interf-labeled entries specify the interference levels seen by client (1,2,3). If each AN chooses the channel with the least amount of interference at its client, the system will enter into an infinite loop – (1,0,0), (1,0,1), (0,0,1), (0,1,1), (0,1,0), (1,1,0), (1,0,0)... – from any initial channel allocation. So while each AN's adaptation process is attractive for a single link, in a network, it is decidedly undesirable.

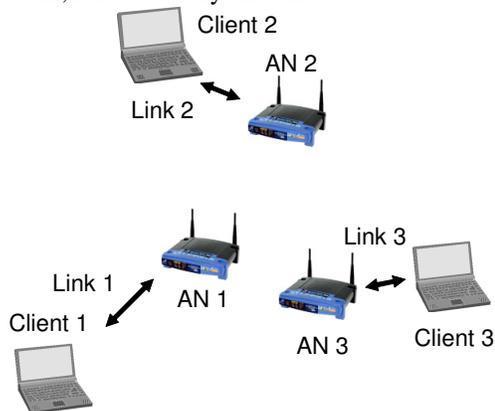


Figure 1: Three Coexisting Uncoordinated Wireless Links

Table 1: Interference Levels for Example DFS Algorithms

Channel	(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)
Interf.	(3,3,3)	(1,2,0)	(2,0,1)	(0,1,2)
Channel	(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)
Interf.	(0,1,2)	(2,0,1)	(1,2,0)	(3,3,3)

Unfortunately, this link gain pattern is not a special case as one out of every four deployments of this system will enter into an infinite loop! In fact, as the number of coexisting links increases, the probability of an infinite loop rapidly approaches 1. Even increasing the number of channels does not eliminate this problem as long as the number of channels is less than the number of adapting links – a seemingly assured situation for practical wireless systems. This example illustrates a critical challenge to deploying CRs – adapting CRs interact network behavior must be considered when designing CR algorithms.

Frequently, this interaction is handled by either using a centralized decision process to control the adaptations of all devices [2,3], by distributing observations throughout the network [4-6], or by explicit coordination [7]. For these

solutions, the spectral advantages presented by adaptive intelligent radios are somewhat muted by the bandwidth consumed to distribute information and coordinate adaptations. However, as we have shown elsewhere [8], it is possible to design uncoordinated CR adaptation processes which use only local (link) information yet converge to the same resource allocations found by centralized or coordinated omniscient algorithms.

In effect, we can have our cake (consume no bandwidth coordinating decisions or distributing information) and eat it too (converge to stable optimal resource allocations)!

Alas, this result cannot be perfectly generalized and our previous publications [8,9] have required networks to satisfy several extremely restrictive conditions. In this paper, we significantly relax these conditions by manipulating the way cognitive radios observe and interpret their local environment to accommodate arbitrary network topologies, much broader classes of waveform parameters, less sophisticated signal processing, and varied levels of quality of service (QoS).

The remainder of this paper is organized as follows. Section 2 presents the theoretical constraint placed on the interrelation of radios' observations which guarantees convergence to a stable optimal resource allocation (bilateral symmetric interference or BSI). Section 3 describes a technique for adjusting observation processes to establish BSI in infrastructure-based networks. Section 4 describes how BSI can be extended to arbitrary network topologies. Section 5 describes a technique which supports differing transmit power levels. Section 6 describes how BSI can be established while providing prioritized QoS. Section 7 concludes this discussion and describes situations where BSI could be immediately applied with great benefit.

## 2. BILATERAL SYMMETRIC INTERFERENCE

In our model of a cognitive radio network, we assume each radio (or link or cluster depending on the context) is controlled by a selfish decision process,  $j$ , which sets the radio's (or link's ...) operating waveform  $\omega_j$  as guided by the selfish goal of minimizing its own observed interference,  $I_j(\omega)$  where  $\omega$  is a vector formed by the waveform choices of all decision processes in the model. Assuming unaltered observations, this goal can be expressed as shown in (1) where  $p_k$  is the transmission power of radio  $k$ ,  $g_{kj}$  is the link gain (path loss) from  $k$  to where  $j$ 's interference observation is made (e.g., a client device in the introductory example) and  $\rho(\omega_j, \omega_k)$  is the absolute value of the correlation between the basis functions of waveforms  $\omega_j$  and  $\omega_k$ .

$$u_j(\omega) = -I_j(\omega) = -\sum_{j \in N \setminus j} g_{kj} p_k \rho(\omega_j, \omega_k) \quad (1)$$

This selfish decision process has the virtue of requiring no coordination or transfer of information between decision processes though as illustrated in the introductory example convergence and optimality are generally not assured.

However, we have shown that when a pairwise symmetry exists between the interference observed by all decision processes in the network, convergence and optimality are assured for all such selfish decision processes [1].

We formally call this condition *bilateral symmetric interference* (BSI) which is satisfied when (2) holds for all possible pairwise subnets formed by only considering decision processes  $j$  and  $k$  (and their controlled radios) where  $\Omega_j$  is the set of waveforms which  $j$  can implement as defined by policy or by device capabilities.

$$I_j(\omega_j, \omega_k) = I_k(\omega_k, \omega_j) \forall (\omega_j, \omega_k) \in \Omega_j \times \Omega_k \quad (2)$$

For example, BSI holds in a network consisting of two cognitive radios,  $j$  and  $k$ , each transmitting at the same power level,  $p$ , with a collection of mutually orthogonal waveforms (i.e.,  $\rho(\omega_j, \omega_k) = 1$  if  $\omega_j = \omega_k$  and  $\rho(\omega_j, \omega_k) = 0$  otherwise). This can be readily verified as channel reciprocity implies that the link gain from  $j$  to  $k$  is the link gain from  $k$  to  $j$  so that  $I_j(\omega) = g_{kj} p_k \rho(\omega_j, \omega_k) = I_k(\omega)$ . How to establish this condition in more general cases is not immediately obvious, but when BSI holds the model's set of decision processes,  $N$ , adaptation space,  $\Omega = \Omega_1 \times \dots \times \Omega_n$ , and goals,  $\{u_j\}$ , form an exact potential game with exact potential function given by (3). [1]

$$V(\omega) = -\sum_{j \in N} \sum_{j < k} I_j(\omega_j, \omega_k) = -\sum_{j \in N} I_j(\omega) / 2 \quad (3)$$

Under some rather broad conditions (all adaptations are self-interested, decision processes are not synchronized, and profitable selfish adaptations are made if available), all selfish processes in a potential game converge to a stable maximizer of its potential function [1]. In our case, this means convergence to a waveform allocation,  $\hat{\omega}$ , which minimizes the sum of observed interference levels.

In general,  $\hat{\omega}$  can only be guaranteed as a local maximizer, but to make a fair comparison, a local maximizer is also all that could be guaranteed of any centralized algorithm assuming polynomial time. In fact, the modeled distributed behavior is identical to a centralized local search algorithm. In general, then, when BSI holds, a distributed selfish cognitive radio network will find solutions as good as those by a centralized algorithm or a distributed omniscient algorithm but without the overhead.

## 3. BSI BY SUBTRACTION

Unfortunately as illustrated in the introductory example, BSI does not generally hold. Nonetheless, it is possible to design observation processes such that BSI holds. For instance, in [8] we presented the dynamic frequency selection (DFS) algorithm for fixed 802.11 networks illustrated in Figure 2. In this system, each access node (AN) chooses an operating channel to minimize its observed interference. However, unlike in the introductory example, interference levels observed by the clients are not included in  $I_j(\omega)$  and are instead based solely on the ANs' observations of the receive

signal power levels of RTS/CTS signals broadcast by other ANs. This combination of conditions has the following effects on the components of  $I_j(\omega)$ :

- $p_k = p_j \forall j, k \in N$  as all RTS/CTS messages are generally broadcast at the same power level;
- $\rho(a_j, a_k) = \rho(a_k, a_j)$  as for channel selection  $\rho$  assumes values of 1 ( $a_k = a_j$ ) and 0 ( $a_k \neq a_j$ ) for orthogonal channels and is also symmetric for non-orthogonal channels [10]
- $g_{kj} = g_{jk}$  as for any given frequency in a fixed network the link gain from device  $j$  to device  $k$  is the link gain from device  $k$  to device  $j$ .

So by redefining the observations of (1) to exclude a subset of the interfering signals, we achieve a network where BSI holds. This then assures the convergence of all selfish decision processes guided by  $I_j(\omega)$  to a channel allocation which minimizes sum network interference.

The behavior of such a network is illustrated in a simulation of thirty ANs randomly distributed over 1 km<sup>2</sup> operating in an environment with a path loss exponent of 3, random device placements, and randomly assigned initial channels. The radios are constrained to operate in the eleven channel 5.47-5.725 GHz European upper UNII band (channels 100-140) and to transmit at a common 1 W and assumed to have noise floors of -90 dBm. Figure 3 depicts the transient behavior of the network with the operational channels for each AN (top), perceived interference levels by the ANs (middle), and the sum of observed interference levels (bottom) for the simulated network. Note that sum observed interference is a monotonically decreasing function as predicted by potential game theory.

While the adaptations intend to minimize only AN-to-AN interference, the performance of client devices also dramatically improves with this algorithm. Figure 4 depicts the average reduction in interference levels seen by the clients and access nodes and the average aggregate reduction in AN-to-AN interference (-2V) when twice as many clients are present as ANs and 100 trials performed per number of ANs in the sweep. The ANs were permitted to operate over all 18 channels in the European UNII band and controlled the nearest client devices. For each trial, all positions and all initial channel assignments were randomly assigned. The algorithm yields a greater reduction in interference for the client devices than for the ANs' observed interference for low density deployments with the situation reversed for high density deployments. In all cases, the reduction in the ANs' actual interference (~30 dB for high density) is more than the clients' reduction (~12 dB for high density) and more than the reduction in AN-to-AN interference (~19 dB for high density). In general, similar performance improvements would be expected in a centrally planned network, but now we get the benefit at run-time (post-deployment) without using any bandwidth to coordinate decisions or transmit information between ANs.

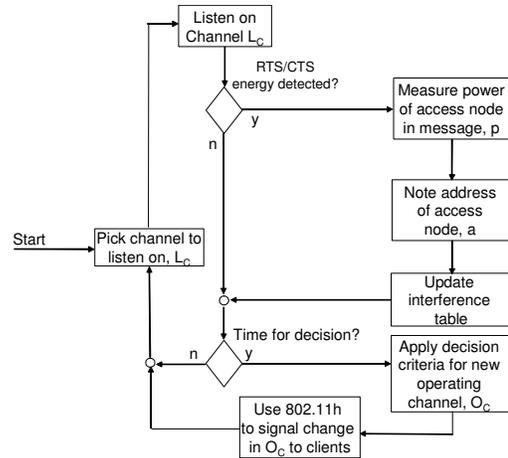


Figure 2: A Simple Non-cooperative Algorithm which Achieves Optimal Frequency Reuse Patterns

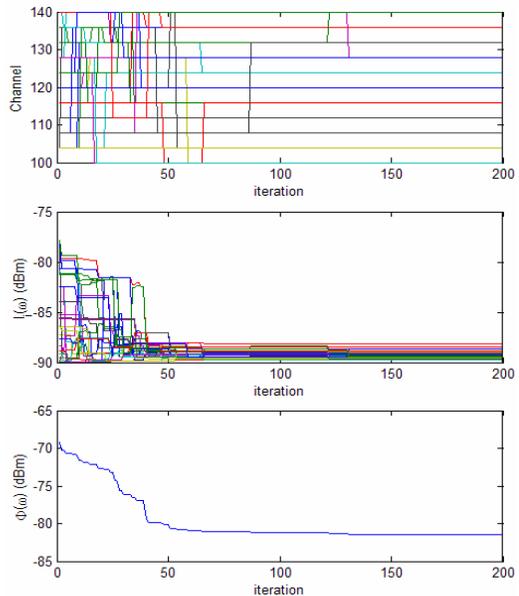


Figure 3: Transient Behavior for a Network Implementing Algorithm Shown in Figure 2.

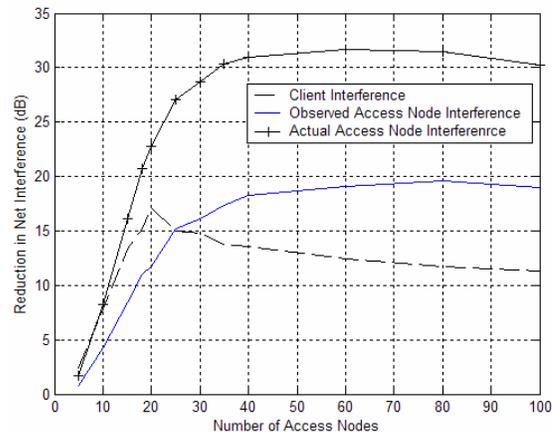


Figure 4: Average Reduction in Interference Levels

#### 4. BSI BY ADDITION (AD-HOC NETWORKS)

In an ad-hoc or peer-to-peer (P2P) network, there is no general topological justification for excluding signals from some nodes and privileging performance of some nodes. So we need a different tack to establish BSI. Consider a system where BSI holds for pairs of devices but decision processes span multiple devices so BSI need not hold, e.g., DFS as applied by a pair of P2P links to signals transmitted with identical power. In such a scenario  $p_k=p_j \forall j, k \in N$  and  $\rho(\omega_j, \omega_k) = \rho(\omega_k, \omega_j)$ , but establishing equal link gains between the decision processes (which span multiple devices) is not as straight-forward.

Now consider the pair of P2P links (1,2) depicted in Figure 5. Assuming BSI holds between pairs of devices, path loss gains between devices are equal. For example, between devices 1a and 2b,  $g_{1a2b} = g_{2b1a}$ . However, the observed interference levels at devices 1a and 2b will be different as they each experience interference from different third parties (2a and 1b, respectively) as shown in (4) and (5).

$$I_{1a}(\omega) = p_2 \rho(\omega_1, \omega_2) [g_{2b1a} + g_{2a1a}] \quad (4)$$

$$I_{2b}(\omega) = p_2 \rho(\omega_1, \omega_2) [g_{1a2b} + g_{1a2b}] \quad (5)$$

But if we form a modified interference observation for the decision process by combining observations from both devices, we get the expressions shown in (6) and (7) which are exactly equal when BSI holds between devices. Thus if BSI holds between all pairs of devices, then BSI between decision processes which span links can be created by having each decision process's observation be the sum of the interference observations of its two devices.

$$I_1(\omega) = I_{1a}(\omega) + I_{1b}(\omega) = p_2 \rho(\omega_1, \omega_2) \sum_{k \in \{a,b\}} \sum_{m \in \{a,b\}} g_{2km} \quad (6)$$

$$I_2(\omega) = I_{2a}(\omega) + I_{2b}(\omega) = p_1 \rho(\omega_2, \omega_1) \sum_{k \in \{a,b\}} \sum_{m \in \{a,b\}} g_{1km} \quad (7)$$

More generally, if we apply this same additive observation technique to a network consisting of any number of wireless clusters with varying number of devices per cluster, then the observed pairwise interference between independently controlled wireless clusters ( $J, K$ ) will be given by (8) and (9) respectively. As (8) always equals (9) when BSI between devices holds,  $I_J(\omega) = I_K(\omega)$  for all pairs of clusters  $J$  and  $K$  in the network thereby establishing BSI.

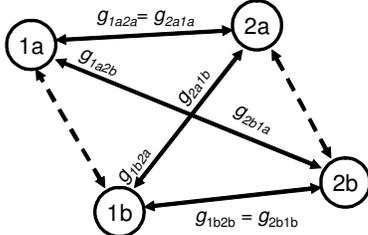


Figure 5: Gains between Devices in Two Arbitrary Links

$$I_J(\omega_j, \omega_k) = \sum_{m \in J} \sum_{n \in K} g_{mn} p_n \rho(\omega_j, \omega_k) \quad (8)$$

$$I_K(\omega_j, \omega_k) = \sum_{n \in K} \sum_{m \in J} g_{nm} p_m \rho(\omega_k, \omega_j) \quad (9)$$

Adjusting the earlier simulation so the system is now a collection of randomly placed P2P links with randomly assigned initial channels operating over a slightly smaller region (0.5 km x 0.5 km), we examined the reduction in interference experienced by selfish uncoordinated links attempting to minimize the observed interference as summed over both devices in its link as expressed in (8). Figure 6 shows the average and typical worst case interference levels for allocations before and after application of the algorithm for varying number of links. Note that the average interference level is reduced by approximately 9 dB and the average of the worst performing links for each simulation is reduced by 30 dB on average. If applied to an 802.11a-like network where collisions only occur when interference levels rise above -82 dBm, Figure 7 depicts the frequency of trials both before and after application of the uncoordinated algorithm which were collision-free. Of interest, it was seen that under a constraint that the system must be collision free 95% of the time, 16 times more links can be supported.

This formulation also provides an interesting secondary benefit. In most systems the observation radius is less than the interference radius [11]. For example in an 802.11 system, signals received below a threshold, (-82 dBm in 802.11a) are not required to trigger a collision event even though interference will still degrade transmission. More generally, a stronger signal is required for decoding than for detection. However, in this formulation the required observation is undesired received signal power in varying bands which can presumably be measured down to the noise floor. This means that for the ad-hoc algorithm, the observable radius is the interference radius. By comparison, the infrastructure based algorithm of Section 3 is dependent on correctly decoding addresses and thus would have differing interference and observation radii.

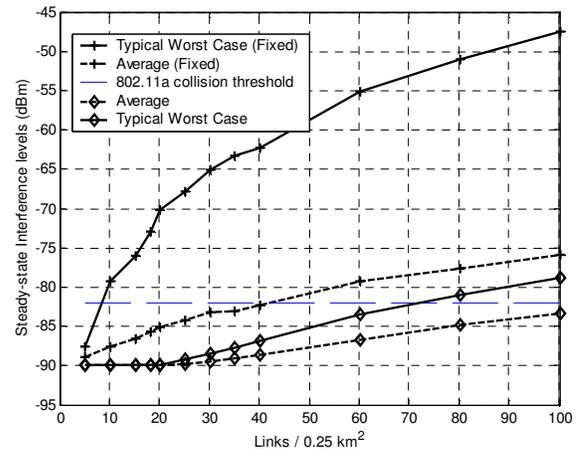


Figure 6: Reduction in Interference Seen in P2P Networks

## 5. BSI BY MULTIPLICATION (BSI FOR UNEQUAL POWER AND PRIORITIZATION)

Consider again the original expression for interference given in (1) and consider the interference induced between two devices  $j$  and  $k$  [ $I_j(\omega, \omega_k) = g_{kj} p_k \rho(\omega, \omega_k)$ ;  $I_k(\omega, \omega_j) = g_{jk} p_j \rho(\omega, \omega_j)$ ]. With unequal powers  $I_j(\omega, \omega_k) \neq I_k(\omega, \omega_j)$  and BSI fails. This is rather disappointing as virtually every wireless system uses power control which means the BSI condition would almost always fail. To overcome this limitation, we can modify the observation processes to achieve the needed symmetry by instructing each device to weight its interference observations by the device's transmit power as shown in (10). Note that  $\tilde{I}_j(\omega_j, \omega_k) = \tilde{I}_k(\omega_k, \omega_j)$  as  $p_j p_k = p_k p_j$ .

$$\tilde{I}_j(\omega) = p_j I_j(\omega) = p_j \sum_{k \in N \setminus j} g_{kj} p_k \rho(\omega_j, \omega_k) \quad (10)$$

This is a relatively costless operation as a cognitive radio should have access to its own transmit power level which means this adjustment only requires no additional bandwidth and only a single multiplication which is independent of the number of devices. Further, scalar multiplication is an order preserving transformation so that whatever channel would have been observed as having the least (most, second most...) interference using  $I_j(\omega)$  will also have the least (most, second most...) interference when using  $\tilde{I}_j(\omega)$ . So to a certain extent, we are still minimizing what we want to minimize (interference) even though the implicit global function being maximized is now (11) (negated weighted interference) instead of (3).

$$V(\omega) = -\sum_{j \in N} \tilde{I}_j(\omega) / 2 = -\sum_{j \in N} p_j I_j(\omega) / 2 \quad (11)$$

Somewhat lost in the preceding is that (10) should not be used as the objective for a decision process which sets transmission power levels as the system will rapidly converge to a state where no device transmit ( $p_k = 0 \forall k$ ). Thus, the power level should be set according to a different objective. However, there will typically be an interaction between the processes which set the waveforms and the power level (e.g., when power is influenced by SINR) and techniques beyond the scope of this paper need to be employed to ensure that infinite adaptation cycles do not arise from this interaction. Nonetheless, when techniques to resolve this interaction are applied significant further reductions in interference are possible as shown in Figure 8 where all links adjust power levels to maintain an SNR of 16 dB (needed for a BER of  $10^{-5}$  for 64 QAM). Note that with the addition of power control, even the worst case interference + noise level barely rises above the noise floor.

Another apparent limitation to applying the BSI concept to current networks is supporting differentiated or prioritized access, e.g., giving higher data rates or better channel access to different users. However, if we assume that the BSI condition already holds between all pairs of devices, then

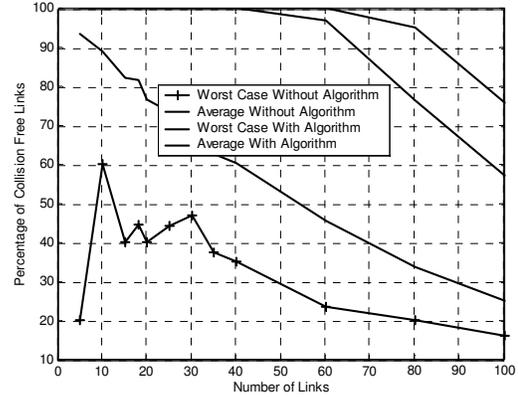


Figure 7: Percentage of Collision Free Trials

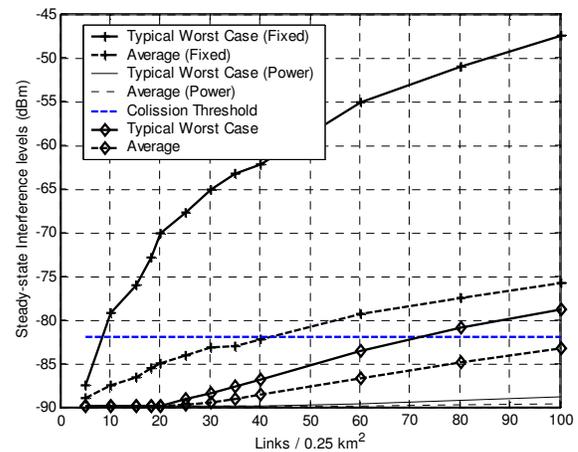


Figure 8: Interference Levels for with Non-cooperative Joint Power-Frequency P2P Networks of Varying Sizes

prioritized access can be supported by announcing to the network weights (priorities) for all devices and altering the observation functions as shown in (12) where  $w_k$  is the weight assigned to user  $k$ .

$$\tilde{I}_j(\omega) = w_j \sum_{k \in N \setminus j} w_k g_{kj} p_k \rho(\omega_j, \omega_k) \quad (12)$$

To evaluate (12), whenever device  $j$  measures received interference power, if  $j$  is able to decode the signal and identify that it is a transmission of device  $k$ , then  $j$  weights the received power by  $w_k$ . Device  $j$ 's aggregate interference observation is then weighted  $w_j$ . This causes the network to converge to resource allocations such that devices with larger  $w_k$  values experience less interference. An example of this phenomenon is shown in Figure 9 where three P2P links are assigned weights of 100 while the remaining 97 links are assigned weights of 1. This results in the 97 links crowding into 15 channels while the 3 links operate in 3 interference-free channels. (To achieve quicker convergence, a round-robin timing scheme was imposed on the network.)

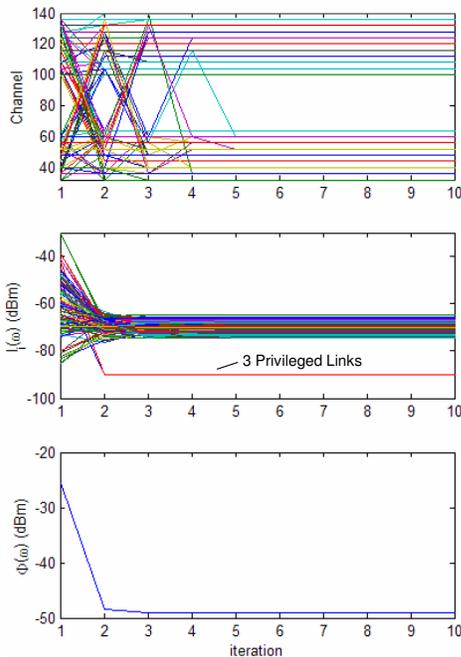


Figure 9: Transient Behavior of a P2P Network with 3 out of 100 Links Weighted by a Factor of 100.

## 6. CONCLUSIONS

When designing a cognitive radio network, it is important to consider how deployed cognitive processes will interact in a network. Without sufficient planning, the interaction of even relatively benign-looking algorithms can yield undesirable behavior. However, by ensuring the network's observation processes satisfy the bilateral symmetric interference (BSI) condition, relatively unsophisticated distributed selfish algorithms will converge to optimal radio resource allocations. In effect, BSI enables cognitive radio networks to achieve the performance of a centralized algorithm with the simplicity of a distributed, selfish algorithm. In general, BSI between pairs of equal power devices occurs naturally as  $\rho(a_j, a_k) = \rho(a_k, a_j)$  for most waveform sets (this need not hold for multiple antenna waveform adaptations e.g., beam forming or MIMO systems). In this paper, we presented techniques for extending the BSI condition to cognitive radio designs intended for arbitrary network topologies or operating with different transmit power levels or for networks which prioritize the traffic of different users. Beyond what is presented in this paper, CRT has also developed low-complexity noncooperative techniques based on the BSI condition for multiple antenna systems, for when internal parameters should be included in the decision process (e.g., battery life), for multicarrier systems (e.g., OFDM), for when policies vary by device, and for when different clusters' activity levels should be considered.

Because CRT has further developed techniques for arbitrarily combining these conditions, virtually any network running virtually any waveform could allocate its layer 1 and layer 2 parameters at run-time using low-complexity non-cooperative distributed algorithms while achieving interference levels equivalent to those realized by centrally planned or massively cooperative algorithms. Using this approach, a metropolitan WLAN network could automatically adapt itself post deployment and as various private networks are deployed and move rather than engaging in extensive pre-planning. Femtocells could be left to sort out their own allocation decisions instead of consuming network resources. Sensors could be arbitrarily dropped without priors or collaboration and still form optimal networks. Network management tasks could be greatly curtailed freeing up personnel for other activities. In the military space, MANETs such as those envisioned by WAND can be greatly simplified by using BSI-based distributed processes for layer 1 and 2 decisions while layer 3 and policy decisions are handled by other processes. Similarly, networks where nodes adjust their positions to improve communications (such as is envisioned in LANDroids) could also benefit from these BSI-based techniques.

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