CARRIER PER CARRIER ANALYSIS OF SDR SIGNALS POWER RATIO

Sidkieta ZABRE[1], Jacques PALICOT[1], Yves LOUET[1], Christophe MOY[1], Christian LEREAU[2]

[1]IETR/Supelec-SCEE Laboratory, Campus of Rennes, Avenue de la Boulais, 35511 Cesson-Sévigné, France
{sidkieta.zabre, jacques.palicot, yves.louet ,christophe.moy}@supelec.fr
[2] France Telecom RD-TECH-GRE, 28 Chemin du Vieux Chêne BP 98 38243 Meylan, France
christian.lereau@francetelecom.com

ABSTRACT

SDR technology proposes very promising solutions to the issue of multi-standard systems design. SDR signals have the particularity to be made of the combination of either several standards and/or different carriers in a single standard at separate frequencies, what implies high temporal fluctuations of the signal. Thus analog non linear elements (such as amplifiers and converters), combined with these high variations, provoke distortions of the signal and consequently a decrease of the system performances (in and out of band distortion, degraded bit-error-rate, etc.). This phenomenon, well known as the PAPR (Peak-to-Average Power Ratio) or simply Power Ratio (PR), is usually addressed in the time domain. This paper proposes a PR frequency approach, which is more adequate to the SDR signal context. By showing the equivalence between the time and frequency approaches for an OFDM signal, we conclude of the interest of studying the PR in the frequency domain, i.e. carrier per carrier, for SDR applications.

1. INTRODUCTION

Over the last few years there has been an explosion in the number of standards, of networks, of services on these networks and finally an exponential increase in bit–rate. This situation explains, partly, why optimized use of different systems and standards as well as efficient spectrum utilization, have become vital research issues.

To the above ends, many different techniques and methodologies are currently studied and a lot of research effort is being invested in the telecommunications area. Of specific interest in this paper, are Software Defined Radio techniques (SDR) and modulation schemes that exhibit high spectral efficiency, such as Orthogonal Frequency Division Multiplexing (OFDM).

An important problem, which limits the performance of the above-mentioned systems, is the presence of non-linearities in the overall communication channel. It is a well-established fact that, in a digital communication system, several elements are sources of non-linearities (mixer, low noise amplifier, power amplifier, analog to digital converter, etc). The most important is the non-linearity associated with the power amplifier in the transmitter, located in the Base-Station or Mobile Terminal. In the applications of interest, the power amplifier, wherever located, will necessarily amplify signals with high Power Ratio (PR) as defined in [1]. However, a power amplifier is commonly characterized by a nonlinear input/output voltage characteristic both in amplitude and in phase. In particular, effective amplification of signals with high PR will be analyzed through the PR parameter.

Note that in both OFDM-based and SDR systems, the involved PR is quite high due to the fact that the transmitted signal is a sum of a large number of modulated carriers. Moreover, a reconfigurable and multifunctional transmitter, as in a SDR base station for instance, should be able to amplify single-carrier and multi-carrier modulation signals as well as many non-constant envelope modulation signals.

The above discussion explains why it is so important to analyze the PR parameter in the SDR context. In this paper, we firstly do an analogy between OFDM signals and SDR signals which results in a common PR definition. Then we will go further by a carrier per carrier interpretation of the PR parameter. By the way we find that well known theoretical upper bound of OFDM signals is applicable to a multiplex of GSM modulated carriers. Furthermore, this vision permits to derive a new upper bound, closest to the realistic one. This result will be illustrated by simulation.

2. SDR SIGNALS FORMULATION

2.1. OFDM Signal

OFDM is a multi-carrier modulation technique which has been adopted by several high data-rate wireless communication standards. In OFDM systems, the complex symbols (from any QAM modulation) are processed through an IFFT to form an OFDM symbol of N carriers. Each OFDM symbol is associated with a corresponding OFDM signal with a duration T_s.

The complex baseband expression of the OFDM signal is given by:

$$Y(t) = \sum_{j=-\infty}^{+\infty} \sum_{k=0}^{N-1} C_{j,k} e^{2i\pi \frac{k}{T_s} t} g(t - jT_s) \quad (1)$$

where g(t) is a rectangular pulse of duration T_s and $C_{j,k}$ represents the complex symbols of the k^{th} carriers of the j^{th} OFDM symbol.
For one OFDM symbol, the baseband signal is given by:

$$Z(t)=\sum_{k=0}^{N-1} C_k e^{2i\pi f_k t} \qquad (2)$$

with $f_k = k/T_S$.

2.2. SDR Signal

A SDR can be defined as a system able of modulating or demodulating any kind of signal, anywhere, on any network. This is of particular value for multi-standard systems. In this context, it is clear that a SDR signal is a composite signal. It is given by this equation:

$$x(t)=\sum_{i=1}^{S} S_i(t) \qquad (3)$$

where $S_i(t)$ represents the i^{th} standard signal and S, the number of standards contained in the composite signal.
The generic equation of a standard, for channels spread on a plurality of carriers, gives:

$$S_i(t)=\sum_{p=1}^{P_i} r_{i,p}(t)\, e^{2i\pi f_{i,p} t} \qquad (4)$$

where $r_{i,p}(t)$ represents the modulated and filtered signal associated to the carrier p of the standard i, which is at the f_i frequency. Then we find the following equation:

$$x(t)=\sum_{i=1}^{S}\sum_{p=1}^{P_i} r_{i,p}(t)\, e^{2i\pi f_{i,p} t} \qquad (5)$$

with $r_{i,p}(t) = fem_i(t) * m_{i,p}(c(t))$
where $m_{i,p}(t)$ represents the modulation relative to the carrier p and $fem_i(t)$ the pulse shaping filter function. Then, the multi-standard signal equation can be written as below:

$$x(t)=\sum_{i=1}^{S}\sum_{p=1}^{P_i} \left(fem_i(t) * m_{i,p}(c(t))\right) e^{2i\pi f_{i,p} t} \qquad (6)$$

3. ANALOGY BETWEEN OFDM SIGNALS AND SDR SIGNALS

In this section we will analyze the conditions in which OFDM and SDR signals are equivalent. The direct consequence of this equivalence would be the applicability of OFDM PR definition to SDR signals.
According to equation (5), it is clear that if the distance between the carriers would be constant for any standards, we would indeed have one SDR signal, which becomes, by definition, a multi-carrier signal:

$$x(t)=\sum_{i=1}^{S}\sum_{p=1}^{P_i} r_{i,p}(t)\, e^{2i\pi\left(f_0+(p-1)\Delta p\right)t} \qquad (7)$$

Generally speaking, this condition cannot be verified because the distance between carriers is different from one standard to another one. It varies from 25 kHz for the PDC and the ICO to 20 MHz for the Hiperlan, by going through 1.712 MHz for the DAB. So, when the SDR signal contains more than one standard, it can not be a multi-carrier signal. Therefore, we cannot make any connection between an OFDM signal and a SDR signal.
We suppose now that the SDR signal contains only one standard. So, by definition, it is called a single-standard signal. As in a standard, the distance between carriers is constant, a connection can be established with OFDM signals, which is a multi-carrier signal. Consequently, the single-standard signal equation (4) can be written as follows:

$$S_i(t)=\sum_{p=1}^{P_i} r_{i,p}(t)\, e^{2i\pi\left(f_{i,0}+(p-1)\Delta p_i\right)t} \qquad (8)$$

From the analytical point of view, relation (8) is indeed formulated in the same way as equation (2) modelling the OFDM signal. In all other cases, we can not make a connection between SDR signal and OFDM signal. OFDM signals remain a particular case of the SDR signals. This equivalence makes possible the PR analogy between OFDM and single-standard SDR signal.

4. POWER RATIO DEFINITIONS

Numerous versions of Power Ratio exist, expressing the same idea to describe the instantaneous power variations of a signal related to its mean power.
PMEPR (Peak to Mean Envelop Power Ratio), PAPR (Peak to Average Power Ratio), CF (Crest Factor), or PEP (Peak Envelop Power) can be found in the literature ([1], [2], [3], [4]) and make the comprehension of the power ratio phenomena a little complex in spite of the fact that they are very close to each other. In a previous article [1], a general expression, called Power Ratio (PR), has been proposed and which is general enough to be declined in many versions depending on the application.

Thus, several notations has been used to define precisely the conditions in which the PR is derived. The indexes c (for continuous), s (for sampled), *i* (for instantaneous) and *m* (for maximal) can be combined to express the four possible PR situations illustrated in Figure 1:

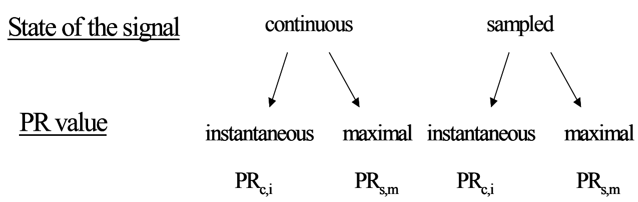

Figure 1 – The 4 continuous and sampled PR expression

In the continuous (theoretical) maximal case, the PR expression is given by:

$$PR_{c,m} = \frac{Max\,|S(t)|^2}{E\{|S(t)|^2\}} \tag{9}$$

where *E* means *expectation*.
In the continuous instantaneous case, the PR is computed over a finished *T* period, the PR expression is given by:

$$PR_{c,i} = \frac{Max_{[0,T]}\,|S(t)|^2}{\frac{1}{T}\int_0^T |S(t)|^2\,dt} \tag{10}$$

In the same manner, we have in the maximal sampled case:

$$PR_{s,m} = \frac{Max\,|s(n)|^2}{E\{|s(n)|^2\}} \tag{11}$$

and in the instantaneous sampled case:

$$PR_{s,i}(N) = \frac{Max\,|s(n)|^2}{\frac{1}{N}\sum_{n=0}^{N}|s(n)|^2} \tag{12}$$

These equations will enable to show the time/frequency equivalence for the PR computation in OFDM and consequently single-standard SDR signals.

5. A NEW APPROACH OF OFDM PR ON SEVERAL SYMBOLS: CARRIER PER CARRIER ANALYSIS

We consider now a new approach where we group N_s OFDM symbols as it could be easily seen in Figure 2. Top Figure 2 represents the numerical complex symbols in the frequency domain and the corresponding samples in the time domain (obtained by IFFT of first table) in bottom Figure 2. Each column of top Figure 2 represents the N_s frequency complex symbols $C_{j,p}$ on carrier f_p. For each carrier f_p a PR in the frequency domain is computed by:

$$PR_{fr}(p) = \frac{Max_{j=1}^{N_s}\left|C_{j,p}\right|^2}{P(C_{1\to N_s}(p))} \tag{13}$$

where $P(C_{1\text{->}N}(p))$ is the mean power calculated line-wise according to the N_s samples of the carrier *p*. Each line of top Figure 2 represents the N complex symbols of the OFDM symbol j. Note that we could also compute a PR for each line, which corresponds to the PR of a OFDM time symbol. Each line of bottom Figure 2 represents the N points FFT of each line of top Figure 2.

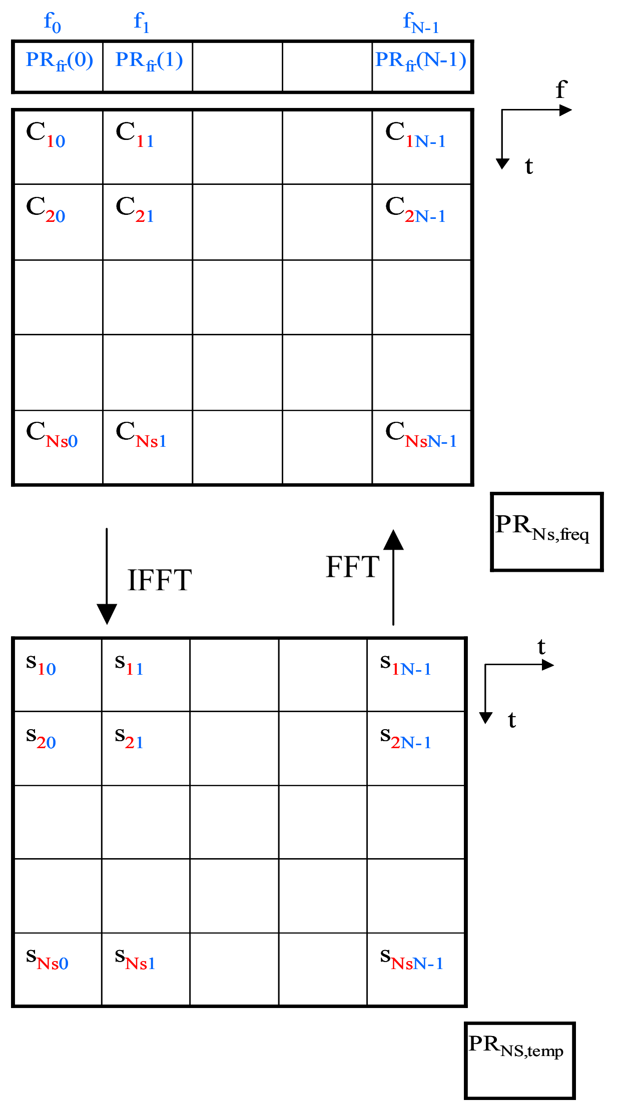

Figure 2 – Time / frequency equivalence of OFDM PR

We deduce for bottom Figure 2 an upper bound of $PR_{Ns,temp}$ from carrier per carrier PR. This improves the previous upper band presented in [5]. Note that we compute the $PR_{s,i}$ from each line of the bottom Figure 2.

5.1. Mean power analysis

Before proceeding further let us define notations used throughout the paper. The mean power of top Figure 2 table is given by the following equation:

$$P_m(C_{1\to N_s}) = \frac{1}{N}\sum_{p=1}^{N} P_m(C_{1\to N_s}(p)) = \frac{1}{N\times N_s}\sum_{k=1}^{N_s}\sum_{p=1}^{N}\left|C_{k,p}\right|^2 \quad (14)$$

with

$$P_m(C_{1\to N_s}(p)) = \frac{1}{N_s}\sum_{k=1}^{N_s}\left|C_{k,p}\right|^2$$

The mean power of top Figure 2 table is given by the following equation:

$$P_m(S_{1\to N_s}) = \frac{1}{N\times N_s}\sum_{k=1}^{N_s}\sum_{p=1}^{N}\left|s_{k,p}\right|^2 \quad (15)$$

Then after some math we propose the following relationship between the computation of the mean power in the time and the frequency domains:

$$\sum_{p=1}^{N} P_m(C_{1\to N_s}(p)) = P_m(S_{1\to N_s}) \quad (16)$$

As already stated in [5] and [6], this relation highlights the equivalence between time and frequency mean power, which is validated by the simulation results of Figure 3. This will help for time PR upper bond with a frequency analysis.

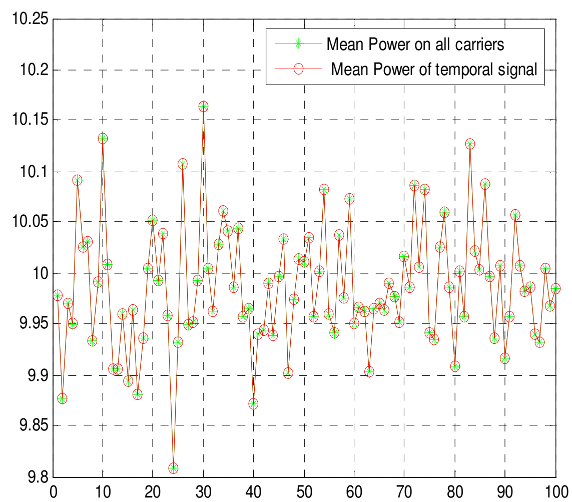

Figure 3 – Frequency and time mean power for 64 OFDM carriers and 100 OFDM symbols (N_s=100).

5.2. Max power analysis

From equation (13), we deduce:

$$Max_{u=1}^{N_S}\left|C_{up}\right|^2 = P_m\left(C_{1\to N_S}(p)\right)PR_{Freq}(p) \quad (17)$$

and as

$$PR_{Temp} = \frac{Max_{u=1}^{N_S}\left(Max_{n=1}^{N_P}\left|S_{un}\right|^2\right)}{P\left(S_{1\to N_S}\right)} \quad (18)$$

It can be shown that

$$\left|S_{un}\right|^2 = \sum_{p=1}^{N_P}\left|C_{up}\right|^2 + \sum_{p=1}^{N_P} C_{up}\sum_{p'\neq p} C_{up}^{*}.e^{-i.2\Pi\frac{(n-1)(p-p')}{N_P}} \quad (19)$$

and

$$PR_{temp} \le \frac{\left(\sum_{p=1}^{N} P_m(C_{1\to N_S}(p))\times PR_{fr}(p)\right)+K}{\sum_{p=1}^{N} P_m(C_{1\to N_S}(p))} \quad (20)$$

where

$$K = Max_{u=1}^{N_s}(Max_{n=1}^{N}\lambda(u,n)) \quad (21)$$

is a constant for each carrier, and

$$\lambda(u,n) = \sum_{p=1}^{N} C_{u,p}\sum_{p\neq p'} C_{u,p'}e^{2.i.\pi.\frac{(n-1)(p-p')}{N}} \quad (22)$$

Let us define

$$A_P = \sum_{p=1}^{N} P_m(C_{1\to N_S}(p)) \quad (23)$$

With this notation equation (20) becomes:

$$PR_{temp} \le \frac{\left(\sum_{p=1}^{N} A_P.PR_{fr}(p)\right)+K}{\sum_{p=1}^{N} A_p} \quad (24)$$

Equation (24) gives the upper bound PR_{temp} of the time PR computed thanks to the frequency PR. In this equation, the PR_{temp} is upper bounded by the addition of the weighted average of the frequency PR and a constant. Figure 4 validates by simulation that the upper-bound follows very closely the PR_{temp} (for 64 OFDM carriers and 100 OFDM symbols).

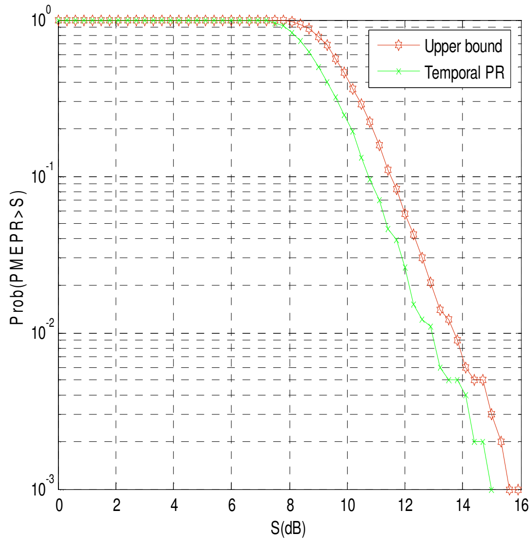

Figure 4 – Comparison of the complementary cumulative distributive function of the time PR and frequency PR.

5.3. GSM

By analogy with what has been just seen for the OFDM and under the hypothesis of regular spacing between the carriers and time and frequency orthogonality, we can then put the single-standard GSM signal in the form of Figure 2. Thus, the same definition as in OFDM can be applied in GSM case. Consequently, the equation (16) and (20) are also true in GSM case. It is exactly what proves the result of Figure 5.

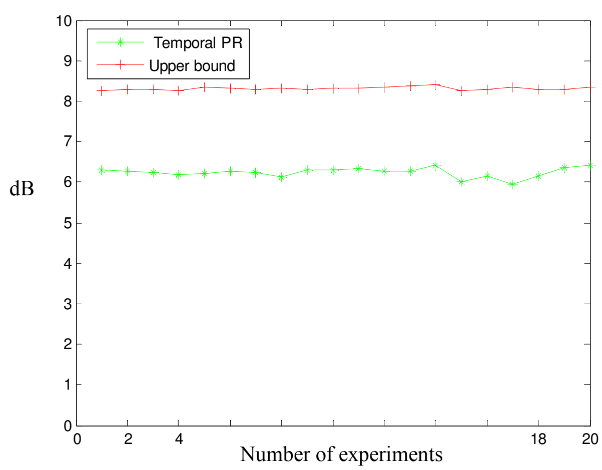

Figure 5 – Time power ratio, and the corresponding upper-bound of (20), for 10 GSM carriers and 20 experiments

In this simulation, 10 GSM carriers have been generated, each of them with 1000 symbols. The time power ratio as well as the inequality of (20) have been computed in Figure 5. As it was expected, the upper-bound given by the power ratio in the frequency domain is close (around 2 dB) to the time power ratio, and follows the same variation. This result illustrates the instantaneous power ratio over 20 runs, but this should be extended to statistical results such as those of Figure 4 in the OFDM case. We expect to extend this result to any kind of SDR single-standard signal: i.e. to derive the time power ratio with its frequency power ratio counter part. This is to be linked with the results of [7] showing that one can create and demodulate GSM signals in the frequency domain.

6. CONCLUSION

Ideal software radio would aim at digitizing the total radio bandwidth with very high speed AtoD converters. Due to technological constraints, SDR limits the digitization to a subset of the total spectrum. If this is limited to the total spectrum of a standard (all the frequency channels of this standard), we have revealed an analogy between the obtained SDR signal and a multi-carrier modulation.

Consequently the high Power Ratio of OFDM signals, which can result in significant signal distortion in presence of nonlinear amplifiers, is also a main drawback of SDR signals. This means that all the methods and analysis applied to the OFDM signals concerning the PR issue are applicable to the SDR signals. Before studying techniques to decrease the effects of non-linearities, it is worthwhile and necessary to clarify the Power Ratio notion in order to qualify the performances of all new techniques studied in the SDR context. It is what we have initiated in this paper. A carrier per carrier analysis of multi-carrier signals (SDR or OFDM) power ratio is derived, which has been illustrated by simulation results.

7. REFERENCES

[1] Yves Louet, Jacques Palicot, "Power Ratio Definitions and Analysis in Single Carrier Modulation", *EUSIPCO'05*, Sept 2005

[2] Mathias Friese, "Multi-tone Signals with Low Crest Factor", IEEE Transaction on Communication, Vol. 45, No. 10, October 1997.

[3] Hideki Ochiai, "On the distribution of the Peak-to-Average-Power-Ratio in OFDM signal ", IEEE Transaction on Communication, Vol. 49, No. 2, February 2001.

[4] Richard Van Nee, Ramjee Prasad, " OFDM for wireless multimedia communications", universal personal communications Artech House publishers, Chapter 6.

[5] Sidkieta Zabré, Jacques Palicot, "Analyse du Facteur de Crête dans le contexte Radio Logiciel ", GRETSI, Juin 2005.

[6] Sidkieta Zabré, Jacques Palicot, " PAPR Analysis of a multiplex of modulated carriers in a Software Radio context ", 5th Karlsruhe Workshop on Software Radio, Feb 2004.

[7] Tim Hentschel, "Channelization for Software Defined Base Station", Annals of Telecom, 57, n°5-6, May and June.