Generalized System Model of an Overlay Environment

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Abstract— **Due to the natural limitation of the resource 'spectrum' and an increasing need for wireless communications, it is indispensable to increase the efficiency of the spectrums utilization. One approach to achieve this goal is the concept of 'spectrum pooling' [1] which allows the coexistence of two independent systems in the same frequency band, also called spectrum pool.**

The overlay system may only use the time-frequency gaps that are not used by the licensed system. Since the licensed systems allocation is time variant, situations can occur where there are not enough resources left for the overlay system to operate efficiently. In this paper we assume that there are several independent spectrum pools with different allocation characteristics. The overlay system now has to observe the spectrum pools and choose the most suitable pool for its transmission.

I. Introduction

Due to the natural limitation of the resource 'spectrum' and an increasing need for wireless communications, it is indispensable to increase the efficiency of the spectrums utilization. To achieve this goal different proposals exist [2], [3]. One is the concept of 'spectrum pooling' [1] which allows the coexistence of two independent systems in the same frequency band, also called spectrum pool. The licensed system has privileged access to the spectrum, but does not use all available frequency channels and time slots all the time. The overlay system has to monitor continuously the licensed system's frequency allocations and may use only the resulting gaps in the frequency-time domain for transmission.

Usually the allocation of a licensed system changes with time. There are situations where there are a lot of ongoing transmissions, as well as there are situations with very little traffic. A high traffic load in the licensed system results in a lot of allocated spectrum, leaving only little resources for an overlay system. At some point the overlay system cannot operate correctly anymore.

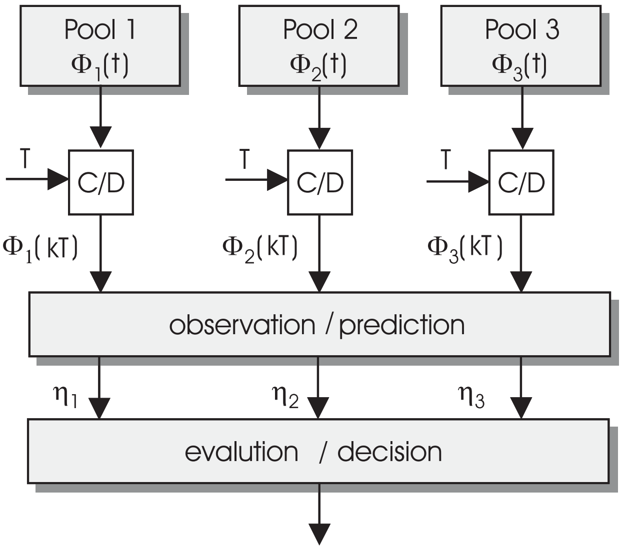


Fig. 1. Generalized system model.

On the other side it might be possible that there is enough free spectrum available in a different frequency band. An optimum overlay system would monitor several frequency bands in order to switch to the most suitable one.

The remainder of the paper is organized as follows. In section II the assumed overlay environment is described. Section III gives an overview of the generalized system model. The succeeding sections IV to VI focus on the specific blocks of the system model: The spectrum pool allocation model (section IV), observation and prediction (section V) and finally the evaluation and decision block (section VI). A conclusion of the paper is given in section VII.

II. Overlay Environment

In this paper we consider an overlay scenario containing J licensed systems based on a combination of TDMA (time division multiple access) and FDMA (frequency division multiple access) operating in different frequency bands. Each licensed system is regarded as a spectrum pool. We assume

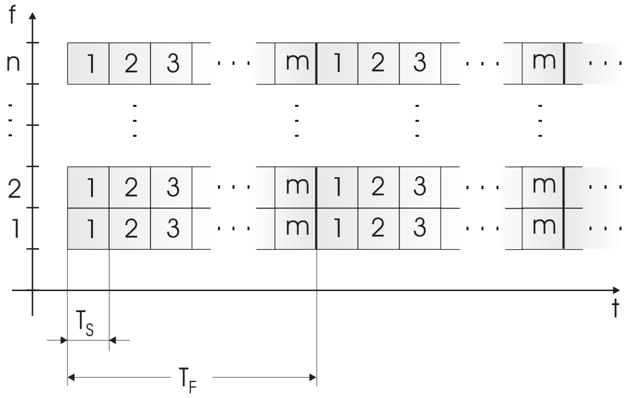


Fig. 2. Spectrum pool based on FDMA and TDMA.

that these systems don't allocate the spectrum continuously which results in the occurrence of time-frequency gaps. To increase the spectral efficiency, these gaps are used for transmissions performed by an independent overlay system that is assumed to be only able to operate in one spectrum pool at a time. Nevertheless it can switch to different spectrum pools depending on defined rules.

In [1] it is shown that OFDM (orthogonal frequency division multiplex) is a suitable technology to utilize these gaps for an overlay transmission. Since the licensed system must not be disturbed in it's operation and there is no information exchanged between both systems, the overlay system has to regularly perform spectrum measurements in the currently used pool. The interval of these measurements shall be denoted by τ. Depending on the licensed system's allocation the overlay system uses a changing subset of the available OFDM subcarriers. The used configuration is described by a so-called allocation vector $\mathbf{v}_j(t)$ whose length equals the number of subcarriers used in the overlay system. The elements of $\mathbf{v}_j(t)$ are binary, where 0 denotes that the specific subcarrier is occupied by the licensed system and therefore must not be used by the overlay system. Accordingly, subcarriers tagged with 1 are available for an overlay transmission.

In order to get an overview on the utilization of the currently unused pools, additional measurements in the interval T with $T >> \tau$ are performed in all pools, e.g. using a wideband-FFT or by sweeping into the different frequency bands.

III. GENERALIZED SYSTEM MODEL

The total amount of available spectrum to the system is grouped into various pools. The goal is to find the spectrum pool fitting best the current needs of the overlay system. In the following we describe

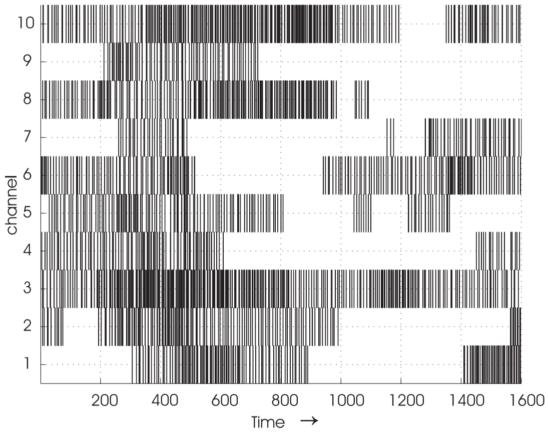


Fig. 3. Allocation of a spectrum pool with $n = 10$, $m = 8$, $\mu = 0.025$, $\lambda = 0.0225$

the concept of a generalized system model targeting this goal. This open framework can be adapted to specific situations. The components are illustrated by examples.

Figure 1 depicts the system model. There are $J = 3$ spectrum pools with a time dependent spectrum allocation. The unused spectrum capacity of the j^{th} pool can be modeled by a time-continuous stochastic process $\Phi_j(t)$, whereas each process can have different characteristics depending on the licensed system operating in each spectrum pool.

Since the overlay system cannot operate in more than one spectrum pool simultaneously and only gets an update of all allocation vectors every T seconds, $\Phi_j(t)$ can be viewed as a discrete function $\Phi_j(kT)$. Based on the observation of $\Phi_j(kT)$ the overlay system can estimate and predict a set of characteristic parameters describing the spectral utilization in the j^{th} spectrum pool. This can be done by different estimation techniques, e. g. by using a Kalman filter.

The resulting set of parameters is denoted by the vector $\boldsymbol{\eta}_j$. The parameters are then evaluated according to the requirements given by an outstanding overlay transmission. Based on the evaluated parameters the overlay system can now choose the most suitable spectrum pool for the overlay transmission.

IV. SPECTRUM POOL ALLOCATION MODEL

In general the allocation of each pool can be arbitrary. In this paper we use a model for the spectrum pools as described in the following. Each pool consists of n channels divided into frames containing m timeslots of the length T_S resulting in the structure shown in Figure 2. T_F denotes the

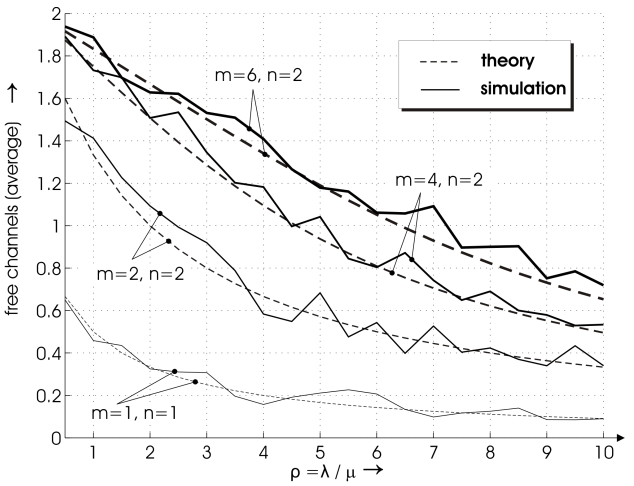


Fig. 4. Average number of free channels depending on ρ.

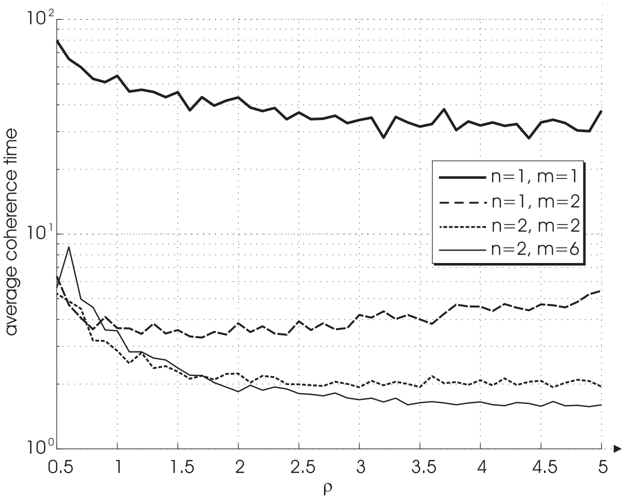


Fig. 5. Average coherence time depending on ρ.

length of a frame. Furthermore we assume a licensed system mainly being used for speech transmissions. The incoming calls can be described by a Poisson process

$$P\{X(t)=i\}=\frac{(\lambda_P t)^i}{i!}e^{-\lambda_P t} \tag{1}$$

with the arrival rate λ_P and $X(t)$ giving the number of calls arriving within the interval t. In order to calculate the unused spectrum we will consider a single channel in the first step. The model will then be extended to multiple channels.

A. Single Channel $(n=1)$

A channel with m timeslots per frame can be modelled as a $M/M/m/m$ queuing system since it can handle up to m calls in parallel. The steady-state probabilities $\boldsymbol{\pi}=[\pi_0,\pi_1,...,\pi_m]$ of this system are given by [4]

$$\pi_i=\pi_0\left(\frac{\lambda}{\mu}\right)^i\frac{1}{i!}, i=1,2,...,m \tag{2}$$

with

$$\pi_0=\left[\sum_{i=0}^{m}\left(\frac{\lambda}{\mu}\right)^i\frac{1}{i!}\right]^{-1}. \tag{3}$$

λ is the arrival rate at the channel and μ the service rate. π_1 e. g. gives the probability that exactly one call is taking place and thus only one timeslot per frame is occupied. Provided that the used timeslot is chosen randomly in every frame and $\frac{1}{\mu}\gg T_F$, the probability for a specific timeslot being occupied is given by

$$p_b=\sum_{i=0}^{m}\frac{i}{m}\pi_i, \tag{4}$$

and

$$p_f=\sum_{i=0}^{m}\frac{m-i}{m}\pi_i, \tag{5}$$

respectively for a free timeslot. With equations (2) and (3) we get

$$p_f=\frac{\sum_{i=0}^{m}\frac{m-i}{m}\left(\frac{\lambda}{\mu}\right)^i\frac{1}{i!}}{\sum_{i=0}^{m}\left(\frac{\lambda}{\mu}\right)^i\frac{1}{i!}}. \tag{6}$$

Equation (6) gives the probability that a specific timeslot is available for an overlay transmission.

B. Multiple Channels $(n>1)$

Considering multiple channels, the total number of incoming calls at the pool is distributed equally among the channels. Hence, the arrival rate at each channel is

$$\lambda=\frac{\lambda_G}{n}. \tag{7}$$

We are now interested in the number of simultaneous free timeslots Y which leads to a binomial distribution

$$P(Y=l)=\binom{n}{l}p_f^l(1-p_f)^{n-l} \tag{8}$$

with the expectation

$$E(Y)=np_f=n\frac{\sum_{i=0}^{m}\frac{m-i}{m}\left(\frac{\lambda}{\mu}\right)^i\frac{1}{i!}}{\sum_{i=0}^{m}\left(\frac{\lambda}{\mu}\right)^i\frac{1}{i!}} \tag{9}$$

and the variance

$$D^2(Y)=np_f(1-p_f)=$$

$$n\frac{\sum_{i=0}^{m}\frac{m-i}{m}\left(\frac{\lambda}{\mu}\right)^{i}\frac{1}{i!}}{\sum_{i=0}^{m}\left(\frac{\lambda}{\mu}\right)^{i}\frac{1}{i!}}\cdot\left(1-\frac{\sum_{i=0}^{m}\frac{m-i}{m}\left(\frac{\lambda}{\mu}\right)^{i}\frac{1}{i!}}{\sum_{i=0}^{m}\left(\frac{\lambda}{\mu}\right)^{i}\frac{1}{i!}}\right). \tag{10}$$

Following the Moivre-Laplace's limit theorem this distribution converges towards a normal distribution for large n.

An example allocation of the licensed system is shown in Figure 3 with $n = 10$ channels and $m = 8$ timeslots.

V. OBSERVATION AND PREDICTION

The main task of this block is to observe all pools within the complete system in order to estimate and predict relevant parameters needed by the decision block.

A. *Observation*

Since in our model the overlay system can only operate in a single pool at a time, it is necessary to observe the other pools and periodicly perform some measurements of the allocation. This can e. g. be done with a wideband FFT or by sweeping into the different frequency bands. These wideband measurements are performed every T seconds, resulting in a time-discret function of the spectrum allocation in each pool. Instead of performing only one measurement at T, it is also possible to make several measurements clustered close around T. This allows to derive parameters like the coherence time.

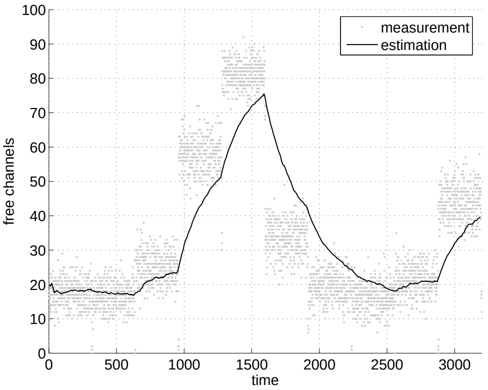
B. *Parameters*

Based on the measurements a parameter vector $\boldsymbol{\eta}_j$ is now derived for each pool j. In this paper we consider the number of free subcarriers $b(kT)$ already described in the previous section and the average coherence time $c(kT)$.

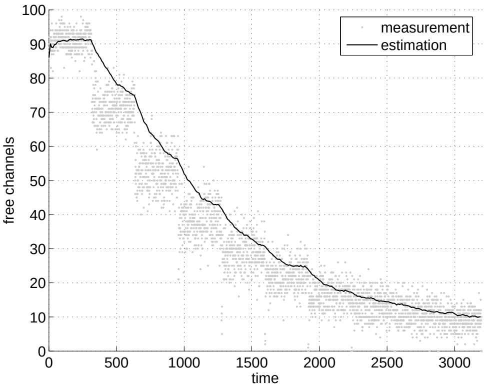
The coherence time is the time during which the pool allocation does not change. This means that the overlay system does not have to update its allocation vector within this time. Note that $c(kT)$ may change even when $b(kT)$ stays constant. Figure 5 shows the average coherence time for different combinations of n and m depending on ρ.

C. *Prediction*

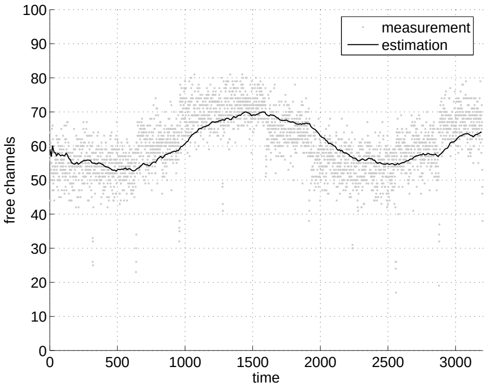
Now that the parameter measurements are available it is necessary to predict the parameters at the next step. This can be done by various algorithms. As an example we used the Kalman filter. Figure 6 shows three pools with $n = 100$ channels. Each pool has a different time dependent allocation with a constant μ but changing λ. The grey dots depict the measurements of the number of free channels. Every $10th$ measurement is used as input to the Kalman filter. The resulting predictions are shown as a black line.



Pool 1



Pool 2



Pool 3

Fig. 6. Allocation measurements and estimation of 3 spectrum pools with changing ρ.

VI. EVALUATION AND DECISION

The predicted set of pool parameters is the input for the evaluation and decision block. Depending on the higher layers different parameters may have a different importance. Therefore, before making a decision which pool provides the best conditions for the current transmission requirements, it is necessary to weight the parameters with a weighting vector w. Based on the weighted parameters the overlay system finally can make a decision whether or not it is better to switch the current spectrum pool.

VII. CONCLUSION

In this paper we considered an overlay environment consisting of several spectrum pools with different spectrum allocation characteristics which are time variant. We presented and examplified a system framework that allows an overlay system to decide which pool currently fits its requirements for a transmission best. In order for the overlay system to choose the most suitable pool it has to observe the spectrum allocation of all pools periodicly and predict a set of relevant parameters.

ACKNOWLEDGMENT

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