COMPLEXITY ANALYSIS FOR KAISER WINDOW AND EQUIRIPPLE PROTOTYPE FILTER DESIGN FOR CHANNELIZATION IDFT FILTERBANKS IN COGNITIVE RADIO APPLICATIONS

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ABSTRACT

Cognitive radio (CR) is gaining widespread interest. One of the key functionalities of a CR device is free channel identification. In this paper, the complexity of the prototype filter in the IDFT filterbanks for the purpose of free channel identification is studied. Specifically, for the Kaiser windowing and equiripple approximation methods, we present the relationship between the number of channels monitored and the prototype filter parameters (number of taps, ripple, and sharpness factor). It is shown that when filter's ripple and sharpness requirements are fixed, the number of filter taps required grows linearly with the number of channels being monitored.

Index Terms—Cognitive radio, IDFT filterbanks, channelization, Kaiser window, equiripple.

1. INTRODUCTION

The current interest in cognitive radio (CR) [1] from the Federal Communications Commission (FCC) is prompting new research directions in both academic and industrial communities. In order for a CR device to function properly, it has to be able to monitor, identify, and utilize frequency channels that are not being fully used by the primary user—adaptive spectrum access, as termed in [2], [3]. This adaptive behavior requires a CR device to possess the ability to examine a wide band of spectrum assigned to the primary users, identify dormant channels, and exploit the available spectrum for its own communications purposes. To achieve this, CR devices need to extract parallel channels from a wideband signal. Extracting parallel channels with identical bandwidth can be achieved easily and efficiently with uniformly modulated filterbanks [4]. It is well known that the combination of polyphase decomposition of prototype filter and inverse discrete Fourier transform (IDFT) provides computationally efficient implementation of uniform modulated filterbanks [5]. This technique simplifies the more complex channelization problem into a straightforward task of designing the finite impulse response (FIR) prototype filter.

Two common practices in designing FIR filters are the windowing of an ideal filter and equiripple approximation using the McClellan-Parks algorithm [9]. For windowing method, the most popular window is the Kaiser method [6] due to the simplicity of Kaiser's empirical formula relating the number of filter taps required with design parameters such as maximum ripple and transition bandwidth. In the context of dormant channel identification for cognitive radio, an additional parameter of consideration that affects the complexity of the prototype filter is the number of channels a CR device desires to monitor, which, as we will see, further constrains the number of taps required for the prototype filter. In this paper, for the Kaiser window method and the equiripple approximation, we will present the number of taps required, L, as a function of M, number of channels being monitored by CR device, δ, maximum ripple of the prototype filter, and S, the sharpness factor of the prototype filter, which we define as the ratio of the passband bandwidth to the transition bandwidth. Furthermore, we show that when δ and S are fixed, L increases linearly with M.

The paper is organized as follows. In section 2, the classical polyphase decomposition and IDFT filterbanks are presented. The required number of taps for the prototype filter when using a Kaiser window is derived in section 3.1 and a design example is given. In section 3.2, a similar analysis is done for the equiripple approximation case along with an example. Finally, concluding remarks are given in section 4.

2. POLYPHASE DECOMPOSITION AND IDFT FILTERBANKS

To take advantage of under-utilized spectrum, a CR device needs to receive a wideband RF signal, digitize the wideband signal, and identify free channels. For example, there are 56 6-MHz TV channels in the 470-800MHz range [7] of which many might be used by a CR device at a specific geographic location. Assume that there are M parallel channels each of bandwidth $2\pi/M$ in the received digitized wideband signal as shown in Figure 1, then to identify free channels, a CR device needs to monitor the energy in each channel and decides that a channel is free to be used if its energy level is below, say the interference temperature as defined by the FCC [8]. Functionally, this requires M parallel filters centering on each of the M individual channels.

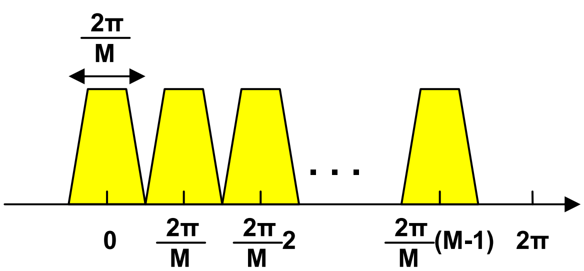

Figure 1 Separate channels in wideband signal

One convenient way to obtain the M parallel filters is to design a lowpass prototype filter [4], $h_0[n]$, with cutoff at π/M, and then shift the prototype filter frequency response by $2\pi k/M$, $1 \le k \le M$-1, to obtain the remaining M-1 bandpass filters. In terms of z-transform, we have

$$H_k(z) = H_0(zW_M^k) \quad 0 \le k \le M-1 \tag{1}$$

where $W_M^k = e^{-j\frac{2\pi}{M}k}$

By definition, the j^{th} branch of an M-polyphase decomposition [6] of the prototype filter, $h_0[n]$, is

$$e_j[n] = h_0[nM + j] \quad 0 \le j \le M-1 \tag{2}$$

Given the M polyphase branches, $h_0[n]$ can be reconstructed as shown in Figure 2.

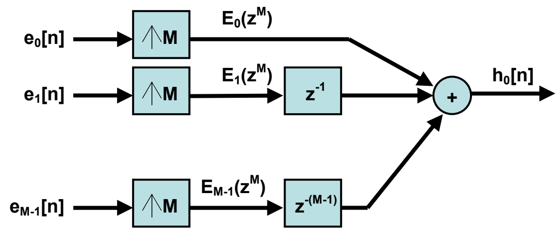

Figure 2 Reconstruction of prototype filter by its polyphase branches

From Figure 2, we can relate the z-transforms of $h_0[n]$ and that of the polyphase branches as follows:

$$H_0(z) = \sum_{j=0}^{M-1} z^{-j} E_j(z^M) \tag{3}$$

Substituting (3) into (1) and rearranging into a matrix form, we have

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ . \\ . \\ H_{M-1}(z) \end{bmatrix} = \begin{bmatrix} 1 & . & 1 \\ 1 & . & W_M^{-(M-1)} \\ . & . & . \\ . & . & . \\ 1 & . & W_M^{-(M-1)^2} \end{bmatrix} \begin{bmatrix} E_0(z^M) \\ z^{-1}E_1(z^M) \\ . \\ . \\ z^{-(M-1)}E_{M-1}(z^M) \end{bmatrix} \tag{4}$$

The M x M matrix in (4) is the inverse discrete Fourier transform (IDFT) matrix. Therefore, we can implement the M parallel filterbanks as M parallel polyphase branches followed by an IDFF. The M signals coming out of the IDFT are highly oversampled, they can be downsampled to reduce the signaling rate of subsequent processing blocks.

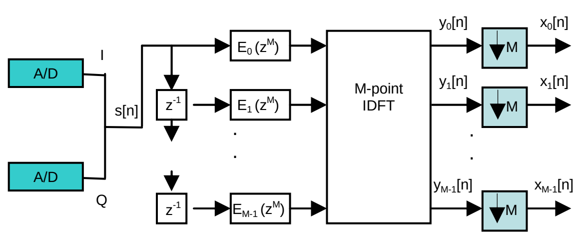

Figure 3 Polyphase decomposition and IDFT implementation for filterbanks

Figure 3 shows the critically sampled [5] filterbank since M is the maximum downsampling rate that avoids aliasing in the downsampling process. The $x[n]$'s in Figure 3 occupy the full spectrum from $-\pi$ and π, and $x_k[n]$ represents the channel centered at $2\pi k/M$. To reduce computational rate and have the polyphase filters operate at the lower sampling frequency, we can exchange the downsamplers and the polyphase branches by applying the Noble Identity, the resulting diagram is shown in Figure 4.

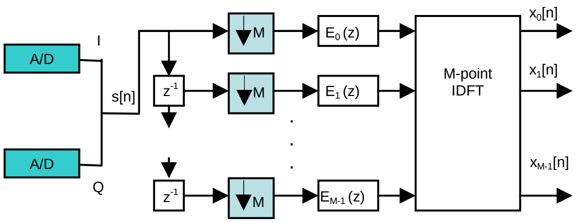

Figure 4 Efficient implementation of filterbanks

The computational complexity of the structure in Figure 4 is the sum of that of the M parallel polyphase branches and that of the IDFT block. Assuming that the prototype filter, $h_0[n]$, has L taps, then L multiplications are needed to implement the polyphase branches; for the M-point IDFT, it is well known the computational complexity is $M\log_2 M$ multiplications. Therefore, the total number of multiplications needed is:

$$L + M\log_2 M \tag{5}$$

3. PROTOTYPE FILTER DESIGN

Once the architecture in Figure 4 is chosen to implement the filterbanks, the next step is to focus on the design of the FIR lowpass prototype filter, $h_0[n]$. There are two main FIR filter design techniques: windowing method and optimum approximations using the McClellan-Parks algorithm [9]. We will discuss both design approaches in the following sections.

3.1. Kaiser window prototype filter design

Kaiser window [6] method for FIR design is widely used due to its simplicity. The two parameters that completely characterize a Kaiser window are:

(1) δ: Maximum passband and stopband ripple
(2) $\Delta\omega$: Transition width of filter

The transition width is defined as

$$\Delta\omega = \omega_s - \omega_p \tag{6}$$

where the passband frequency ω_p of a LPF is defined to be the highest frequency such that $|H(e^{j\omega})| \geq 1-\delta$; and the stopband frequency ω_s is defined to be the lowest frequency such that $|H(e^{j\omega})| \leq \delta$. For the Kaiser window, the number of filter tap required is given in [6], which is repeated here for convenience.

$$L = \frac{-20\log_{10}\delta - 8}{2.285\Delta\omega} \tag{7}$$

It is desirable to relate L with M, the number of channels present in the wideband signal $s[n]$ in Figure 4. We define the sharpness factor of the prototype filter, S, as the ratio of the passband width to the transition width. Therefore

$$S = \frac{\omega_p}{\Delta\omega} \tag{8}$$

The spectrum in Figure 1 dictates that the prototype filter's cutoff frequency should be $\omega_c = \pi/M$. The symmetrical property of the windowing method also implies that ω_c is the midpoint of ω_p and ω_s, therefore

$$\frac{\omega_p + \omega_s}{2} = \frac{\pi}{M} \tag{9}$$

Solving (8) and (9), we obtain, for large S

$$\Delta\omega = \frac{2\pi}{(2S+1)M} \approx \frac{\pi}{SM} \tag{10}$$

Substituting (10) into (7), we have

$$L = \frac{(-20\log_{10}\delta - 8)S}{2.285\pi}M \tag{11}$$

It is clear from (11) that when the maximum ripple and the sharpness of $h_0[n]$ are fixed, the number of taps required increases linearly with the number of channels to be monitored. For M = 8, δ = 0.01, and S = 10, evaluating (11) gives L = 357, which means that $h_0[n]$ will be a Type I filter. If we choose $h_{ideal}[n] = \frac{\sin[\pi/M(n-(L-1)/2)]}{\pi(n-(L-1)/2)}$ as the ideal lowpass filter to be windowed, then the impulse response of the resulting L = 357 tap $h_0[n]$ is shown in Figure 5, along with the amplitude ripples and phase response. Note that in the passband, $h_0[n]$ exhibits linear phase as expected of Type I filters. Notice also that $h_0[n]$ is a causal M^{th} band filter [4] since $h_0[n] = 0$, when $n = (L-1)/2 + kM$, $k=\pm1$, ±2… This implies that one of the polyphase branches in Figure 4 has only a single non-zero tap.

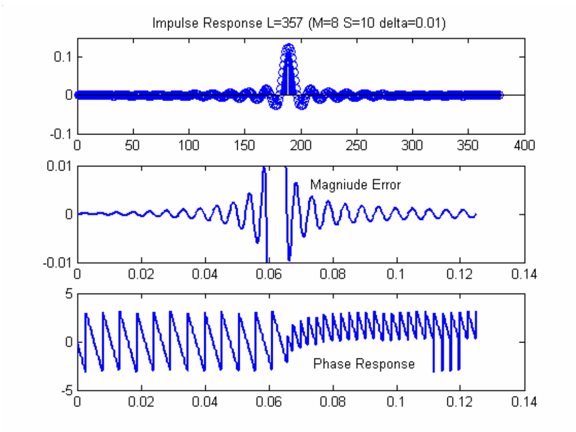

Figure 5 Example--Kaiser window based prototype filter

3.2. Equiripple prototype filter design

Equiripple approximation is an optimal design method in the sense it minimizes the maximum ripples. The widely used Parks-McClellan Algorithm based on the Alternation Theorem is discussed in [6]. For equiripple lowpass approximations, Kaiser obtained the following formula relating *L*, the number of taps, with $\Delta\omega$, transition bandwidth, δ_1, passband ripple, and δ_2, stopband ripple.

$$L = \frac{-10\log 10(\delta_1\delta_2) - 13}{2.324\Delta\omega} \qquad (12)$$

For equal passband and stopband ripple, $\delta_1=\delta_2=\delta$, substitution of (10) into (12) gives

$$L = \frac{(-20\log_{10}\delta - 13)S}{2.324\pi}M \qquad (13)$$

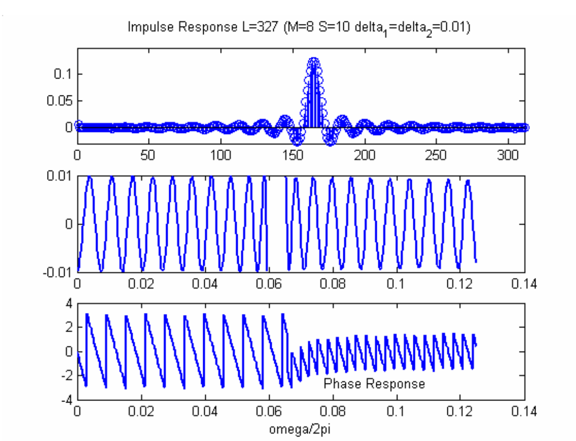

Figure 6 Example—prototype filter based on equiripple approximation

For $M = 8$, $\delta = 0.01$, and $S = 10$, evaluating (13) gives $L \approx 300$.

Figure 6 shows the corresponding filter along with the ripples and phase response.

For the same design specifications, we can compare the Kaiser windowing and the equiripple methods. Figure 7 clear shows that when *S* and δ are fixed, for both methods, the number of taps required of the prototype filter varies linearly with the number of channels. Figure 7 also shows that equiripple method will always produce a shorter filter as compared to Kaiser window method due to its smaller slope.

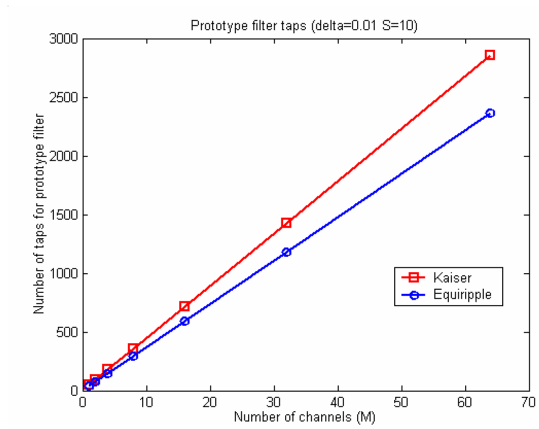

Figure 7 Number of filter taps required for Kaiser window and equiripple method

We can also compare the implementation complexity of using these two design methods. Recall that the total number of multiplications needed to implement the filterbanks is given in (5) for any prototype filter. By substituting (11) into (5) and dividing by *M*, we obtain, for the Kaiser window method, the number of multiplications needed per channel

$$\frac{(-20\log_{10}\delta - 8)S}{2.285\pi} + \log_2 M$$

Similarly, the number of multiplications per channel for the equiripple approximation method is

$$\frac{(-20\log_{10}\delta - 13)S}{2.324\pi} + \log_2 M$$

Figure 8 shows the comparison. Again, due to the smaller number of taps, the equiripple method requires a smaller number of multiplications.

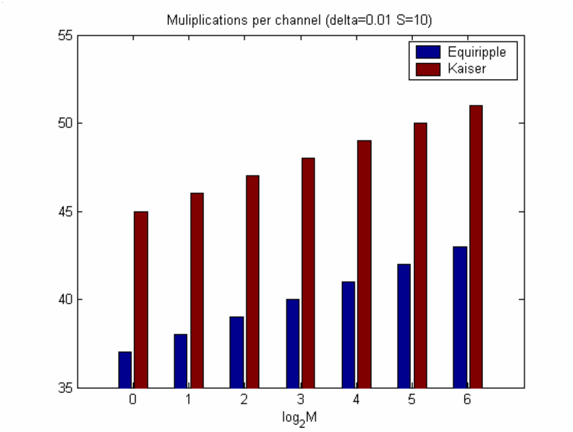

Figure 8 Multiplications per channel needed for Kaiser Window and equiripple prototype filter

4. CONCLUSION

In this paper, we described the prototype filter requirement in the context of using filterbanks and IDFT for cognitive radio channelization. Complexity analyses in terms of number of filter taps needed were given for both Kaiser Window and equiripple approximation methods. A first order analysis for the number of multiplications per channel is also given for the aforementioned two methods.

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